UNIVERSITY OF TORONTO
3 1761 01292127 6

ELEMENTARY ELECTRICITY

BY

S. G. STARLING

B.Sc., A.R.C.Sc., F.Inst.P.

HEAD OF THE DEPARTMENT OF PHYSICS IN THE WEST HAM MUNICIPAL COLLEGE

AUTHOR OF "ELECTRICITY AND MAGNETISM, FOR ADVANCED STUDENTS," "SCIENCE IN THE SERVICE OF MAN: ELECTRICITY," ETC.

NEW IMPRESSION

WITH 216 ILLUSTRATIONS

LONGMANS, GREEN AND CO.

LONDON • NEW YORK • TORONTO

1929

LONGMANS, GREEN AND CO. LTD.

39 PATERNOSTER ROW, LONDON, E.C.4
6 OLD COURT HOUSE STREET, CALCUTTA
53 NICOL ROAD, BOMBAY
167 MOUNT ROAD, MADRAS

LONGMANS, GREEN AND CO.

55 FIFTH AVENUE, NEW YORK
221 EAST 20TH STREET, CHICAGO
TREMONT TEMPLE, BOSTON
210 VICTORIA STREET. TORONTO

LIBRARY
MAY 17 1933
UNIVERSITY OF TORONTO

841185

QC
523
S8

Made in Great Britain. *All rights reserved*

PREFACE

An attempt has been made in this book to represent the subject of electricity in a logical manner, and to keep in view its present-day aspect as well as its history. Magnetism is no longer treated as a separate subject, as modern theory holds that magnetism is an attribute of electricity in motion, that is, of the electric current. As an introduction to the subject one of the most familiar applications of electricity is chosen—the electric incandescent lamp, and from this the rest is developed. Those branches of magnetism which stand apart from the main thread of the treatment of electricity, namely, terrestrial magnetism and the magnetic properties of materials, are relegated to the two last chapters. One of the most difficult conceptions for the student, that of potential, is introduced in the treatment of electricity at rest, that is, electrostatics, as it is here a simple quantity. Its relation to the rate of working in the case of a current then follows easily.

While fitted for classes of students studying electricity with an examination in view, the book is also intended to give them a wide outlook upon the subject in its present state of development. For this reason chapters on electric waves and wireless telegraphy and telephony, electrons and X-rays and radioactivity are added; in fact there is a tendency in modern examinations to introduce these subjects to a small extent in order to make electricity a living rather than a historical subject. Only well-established facts have been included, but these prepare the way for the intensely interesting subject of the modern theories of the constitution of matter which the student may follow later.

The student requires a knowledge of elementary mathematics of the standard of the Matriculation or Intermediate

examinations of the Universities. Examples have been added at the end of each chapter, for class use, or for the student to test his knowledge as he proceeds. The tables of logarithms and trigonometrical functions at the end of the book are added for the purpose of assistance in working the examples. Thanks are due to the Controller of H.M. Stationery Office and Messrs. Macmillan & Co., Ltd., for permission to use these tables.

S. G. STARLING.

WEST HAM,
Sept. 1923.

CONTENTS

TABLES

MAPS

PLATES

PLATE I

(*a*) Photograph by C. T. R. Wilson of the path of a beam of X-rays through air supersaturated with water vapour, showing the cathode or β-ray tracks produced. Magnification $2\frac{1}{2}$ diameters.

(*b*) Photograph by C. T. R. Wilson of the path of a beam of X-rays in air supersaturated with moisture. Magnification 6 diameters.

Facing p. 192.

PLATE II

Coolidge X-Ray tube in lead glass shield.

Photograph taken by placing a piece of incandescent gas mantle in contact with sensitive plate.

Photograph by C. T. R. Wilson of the track of an α particle from radium through air supersaturated with water vapour.

Facing p. 193.

ELEMENTARY ELECTRICITY

CHAPTER I

THE ELECTRIC CURRENT

General knowledge of electricity.—Most people are more or less acquainted with electric currents and their uses. Lighting by electricity, electric bells, the telegraph, telephone, and electric trams and trains, are all so common that few people are ignorant of the use of electricity in some form or other.. But some of the properties of the electric current which contribute to its usefulness are only recognised by those who have given special study to the subject, while other properties are so obvious that they must be seen by the most casual observer. We will make a starting-point for our study by considering some of the more obvious properties of the electric current ; the rest of this book will be concerned with explaining the others.

Electric current produces heating.—The most obvious effect of an electric current is the heating it produces. As evidence of this, think of the ordinary incandescent lamp. A fine filament of wire is enclosed in a glass bulb from which the air has been pumped. When the electric current passes through this filament heat is produced. With a small current the filament is slightly warmed, but as the current is increased the filament gets hotter and will glow at a dull red heat. With further increase in current the filament gets white hot, and in this condition emits the light so usefully employed. The higher the temperature, the whiter is the light emitted and the greater the efficiency of the lamp. But there is a limit to this efficiency, imposed by the fact that the filament is destroyed if the temperature is too high. If the lamp is of the old carbon filament type, the carbon is disintegrated

and deposited on the glass bulb, which is thus blackened; but if the lamp is of the metal filament type, the filament may be melted. In Fig. 1 is seen a metal filament lamp of a common type. The filament is a fine wire of tungsten carried by the wire spiders B and C. The wires of C are flexible and springy to keep the filament tight at all temperatures, and are carried by the glass pillar A. The metal filament can be used at a higher temperature than the carbon filament, and hence its higher efficiency. Lamps are now made with the bulb partly filled with nitrogen, which allows a higher temperature to be used than with the vacuum lamp. These are called gas-filled lamps.

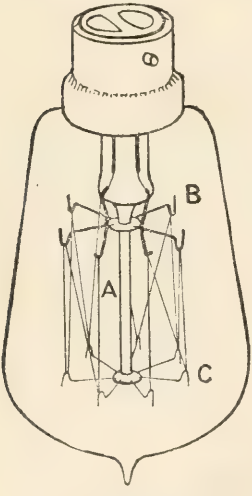


FIG. 1.—Incandescent lamp: metal filament.

The heating produced by an electric current may be observed by connecting the ends of a battery by means of a piece of wire, which will often be found to be warm to the touch. If the wire is sufficiently thin and short it may be rendered red hot and eventually will be melted. This property is commonly used as a safety device in an ordinary house-lighting circuit. At some point of the circuit, a fine wire called a **fuse** is placed, which melts if an excessive current passes, and so breaks the circuit. This leads us to our second point.

The electric current only flows in complete circuits.—In order to light an incandescent lamp, it is always necessary to close a **switch** placed in its circuit. This switch is only a metallic bridge which can be made or broken at will. When it is made, the circuit in which the lamp is situated is completed and the current flows; when broken, the circuit is incomplete and the current ceases. The fuse mentioned above is an instance of the automatic process of breaking the circuit for the purpose of safety if the current becomes too great.

Conductors and insulators.—When we speak of a complete circuit, we mean that there is a continuous path around

which a current can flow. Now the electric current can flow through some materials but not through others. The former are called **conductors** of electricity, and the most important of these are the metals, carbon, and certain liquids, and the most useful of the metals for this purpose is copper. Therefore electric circuits are made of copper wires. For example, the mains which bring the electric current to our houses are thick cables of copper wire, and the wires in the lighting circuit itself are smaller copper wires, providing a complete path from the source of current, which may be the dynamo at the electric lighting station, through the streets, through the wires in the house, the electric lamp itself, and back by another wire and cable to the station. Should there be a break anywhere in the circuit, no current flows.

The other materials, which will not carry an electric current, are called **insulators.** Amongst the most useful of these we may note porcelain, glass, sulphur, and india-rubber. Between the two classes of conductors and insulators there are many substances which conduct the current partially, such as paper, earth, wood and water, and as a general rule the more moist a substance is, the worse will its insulating properties be. Hence to confine the electric current to its proper path, the copper wire cables are surrounded by india-rubber sheaths and the connections, such as switches and fuses, are mounted on porcelain blocks. If this practice were not followed, the current would find many paths other than its proper one. It is a general rule that all electrical apparatus should be kept dry, as otherwise the insulation will become imperfect.

Electric arc.—Amongst the properties of the electric current that are not so well known as the above, the spark produced whenever a circuit is broken may be noticed. This spark may be seen if any circuit carrying a current is interrupted, but unless the current is large the spark is soon quenched. If, however, a sufficiently powerful supply of current is available, and the break in the circuit takes place between two carbon rods, the spark is not soon quenched but may persist for an indefinite time. In this case it is called an **electric arc.** One of the carbon tips is exceedingly bright, its temperature being about 3500° C., far above the melting-point of any known metal. It is of dazzling brightness and forms the source of light known as the arc lamp.

Electrolysis.—Another effect of the current not commonly

known is the occurrence of chemical action when part of the circuit consists of solutions of various substances in water. If two carbon rods dip into a solution of copper sulphate and a current be passed through the solution, one rod will be found to be coated with metallic copper after a short time. On reversing the current, the copper is dissolved from the carbon rod, but is deposited upon the other one. This is an example of the chemical action produced by the current, which is called **electrolysis.** Again, if the carbon rods be dipped into slightly acidulated water, bubbles will arise from the rods when the current passes. On collecting these bubbles of gas, those from one carbon rod are found to be hydrogen and those from the other, oxygen—the constituents of water. The water has been decomposed into its constituent elements by the current. This is another example of electrolysis.

Magnetic effect of a current.—Probably the least well understood, but certainly the most important, property of an electric current is the magnetic effect which it produces.

A simple experiment will illustrate this. If a piece of ordinary insulated copper wire about one metre long be joined to a battery B (Fig. 2), consisting of a few cells of any kind, and the wire be

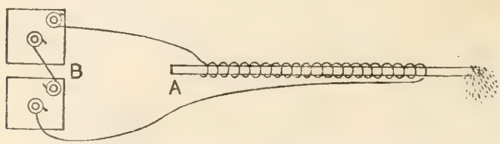


FIG. 2.—Experiment to illustrate the magnetic effect of an electric current.

wrapped round an iron rod A as shown, then on bringing one end of the rod into a heap of iron filings or small iron nails, the filings or nails will become attached in a cluster to either end of the rod. On breaking the circuit anywhere, so that the current ceases, the filings or nails will fall from the rod. We say that the rod is **magnetised** by the current.

This magnetic effect of the current is so important that it must be examined a little further. If the rod A, while the current is flowing, is brought near a magnetic compass, one end of the rod A will attract one end of the compass needle and repel the

other. On using the other end of A the reverse effect is obtained. Also, if the rod be withdrawn, and the wire still left in its spiral form, the same effects will be observed, although they will not be so strong as with the iron rod present. The student is advised to perform this experiment for himself. Even if the wire is straightened out, it will still affect the compass needle, but this is not so easy to follow as the above and will be explained in the next chapter.

Sources of current.—Reference has been made several times to the source of supply of current. The most important source of current is the dynamo, a machine which is driven by a steam or other engine, and is capable of producing the large currents used for public lighting, trains, and other purposes. The understanding of the dynamo involves more knowledge than can be assumed at this stage and must be deferred until later.

Besides the dynamo there are other sources of current of extreme usefulness when only small currents are required for local purposes. These generally take the form of some type of **electric cell.** In the electric cell the energy required to maintain the current is derived from some chemical process occurring in the cell. Just as the chemical process of combustion supplies energy in the form of heat, so it may in the electric cell supply energy in the form of electric current, which may, and frequently does, ultimately take the form of heat. A number of cells may be used together to form a **battery**, when the effect to be produced is greater than one cell alone can accomplish.

There are many forms of cell in common use, amongst which may be noticed the **Daniell's cell** (Fig. 105, p. 124) which gives a small but steady current. Also the **Leclanché cell** (Fig. 107, p. 125) which gives a fair current for a short time only. It is therefore used when intermittent currents are required, with considerable intervals of time between each production of current. Thus for electric bells, telephones, flash-lamps, etc. the Leclanché cell is very useful, especially in its common form, the so-called dry cell. When large currents are to be maintained for lengthy periods, the **secondary cell or accumulator** (Fig. 108, p. 128) must be used. The accumulator, however, unlike the previously mentioned types of cell, only produces current when it has been charged by passing a current through it ; hence its name. These cells all have their particular uses, and their construction and the principles involved will be given in Chapter VIII.

Exercises on Chapter I

1. Describe the heating effect of an electric current and a use to which it is put.

2. Distinguish between conductors and insulators. How would you show that an electric current only flows in complete circuits ?

3. Describe a simple experiment by means of which the magnetic effect of an electric current may be exhibited.

4. Give a short account of the various sources of electric current.

5. Describe some form of electric incandescent lamp in common use.

6. Describe a simple experiment to illustrate the phenomenon of electrolysis.

CHAPTER II

MAGNETS

Permanent magnets.—In the experiment described on p. 4 (Fig. 2) it was observed that when the current circuit was broken, the iron rod ceased to exhibit magnetic effects. If the experiment be performed again, using a steel rod such as a steel knitting-needle in place of the iron rod, it will be found that it becomes a magnet and attracts iron filings as before, when the current flows. The chief difference between the iron and the steel lies in the fact that when the current ceases, the steel knitting-needle retains its magnetic properties, whereas the iron is a magnet only so long as the current flows. A slight difference may be noticed in the readiness with which iron and steel are magnetised. The same coil and current will not magnetise the steel so strongly as the iron; it is therefore advisable to use more turns in the coil when magnetising the steel. Permanent magnets are made in many forms, and when it is desired that they shall retain their magnetisation for very long periods they are made of tungsten steel. Examples of permanent magnets are met with in the case of the compass needle and those of the horseshoe type in Figs. 115 and 130.

Properties of magnets.—It has already been seen (p. 4) that iron filings cling to the ends of a magnet. The observable magnetic effects are confined to the end portions of the magnets. These places where the magnetic effects are obvious are called the **poles of the magnet.**

A very important peculiarity of magnets is, that **they can communicate their magnetic properties to other pieces of iron or steel,** by their mere presence. In fact, the attraction of the magnet for the iron filings implies this property, as will be seen later. In order to magnetise a knitting-needle, it may be stroked from one end to the other with the pole of any fairly strong magnet such

as a bar magnet or a horseshoe magnet. By stroking several times in **the same direction** with the same pole of the bar magnet, the knitting-needle may be made into a fairly strong magnet, which can be tested with iron filings.

In order to discover other properties of the magnet, take the magnetised knitting-needle and support it as shown in Fig. 3 by a thin wire stirrup, W, suspended by a fine silk thread. It will be found that the needle will always come to rest in one direction, and that this direction lies nearly north and south. The end of the needle which always points north is called the north-seeking pole or N pole of the magnet. Similarly the other is the south-seeking or S pole. If the needle has been made by rubbing with a bar magnet, it will be found that the end of the needle at which the rubbing ceases is an **opposite** kind of pole to that with which the needle was rubbed

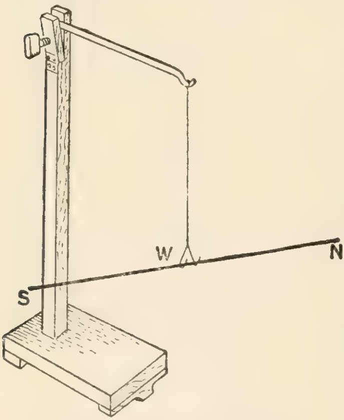


FIG. 3.—Suspended magnet.

Now magnetise a second needle; hang it up to determine its N pole. Using this second needle, examine its effect upon the first needle which is suspended and allowed to come to rest. On bringing the N pole of one needle near the N pole of the other it will be found to repel it. On the other hand it attracts the S pole. Similarly it will be found that the S pole attracts a N pole and repels a S pole. Thus we deduce the rule that **like poles repel each other and unlike poles attract each other.**

Another property of magnets may be found by breaking one of the magnetised needles of the last experiment in two parts, as nearly equal as possible. By testing each half with the iron filings and by bringing it near the suspended magnet. it will be found that two new magnetic poles have appeared, which make each half into a complete magnet having its N and S pole. On further breaking the halves into quarters the same effect will still be found, as represented in Fig. 4. It therefore appears that no magnet can have one pole only, it has always equal N and S poles.

This suggests that every piece of iron or steel consists in its smallest parts of complete magnets. Whether these smallest parts are the molecules or the atoms of iron shall be left an open

question, but there is no doubt that these elementary magnets are free to turn, and that the bringing of a magnet near a piece of iron or steel causes them to turn in accordance with the law of

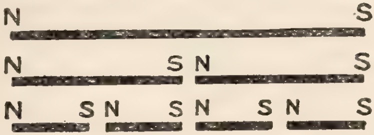


FIG. 4.—Poles produced on breaking a magnet.

force between poles. Thus in Fig. 5 if we represent the elementary magnets diagrammatically by the short arrows, (*a*) represents an unmagnetised piece of iron or steel, with the elementary magnets pointing in all directions, and (*b*) represents the same piece

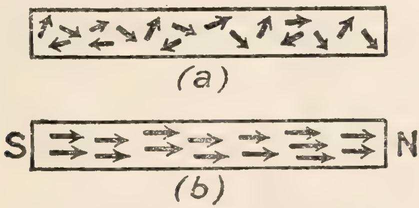


FIG. 5.—Magnetisation of iron or steel.

when a second magnet has been brought near and has caused the N poles all to face to the right, producing a N pole at N and a S pole at S.

Force between poles.—In order to deal with the measurement of electric current, it is necessary to examine the laws governing the force between magnetic poles, for it is by means of the magnetic effect which it exerts that the electric current is measured.

The simple experiment on p. 8 will make it clear that the nearer that two magnetic poles are brought together, the stronger will be the force between them. If, therefore, we can find some method of measuring the strength of a magnetic pole, we can then investigate the law of force between poles. We will in the first place define poles as being of equal strength if, on being brought one after the other into the same place, they all experience the same force. Imagine a number of such poles to be obtained, and one pole made up by taking a number, say m_1, together, and another of a

different strength by taking m_2 of them. Then the force between these two poles will be proportional to the product of their two strengths; thus the force between them is proportional to $m_1 \times m_2$. If the poles are both N or both S the force is of the nature of a repulsion, but if one is N and the other S, the force is an attraction. In Fig. 6 it is assumed that the poles are alike in kind.

The law governing the dependence of the force upon the distance apart cannot be proved accurately by any simple experiment, but it will be stated here, and its reliability will follow from the fact that the calculated results obtained by means of it are always in accord with experiment. The law is—**the force between any two magnetic poles varies inversely as the square of their distance apart.** Thus the complete law of force between magnetic poles may be stated in the form—

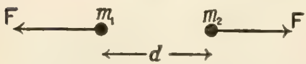


FIG. 6.—Force between poles

$$\text{Force is proportional to } \frac{m_1 m_2}{d^2}$$

where d is the distance between the poles.

Unit magnetic pole.—In all electrical measurements the International or Scientific system of units is adopted. This system takes the centimetre as the unit of length, the gramme as the unit of mass, and the mean solar second as the unit of time. On such a system the unit of force is the **dyne.** A dyne is therefore the force which will produce in a mass of one gramme a velocity of one centimetre per second, if acting for one second.

The force between magnetic poles should therefore be measured in dynes, and in order to obtain a known force between any given magnetic poles, those poles must be measured in suitable units. **We choose our unit magnetic pole as of such a strength that when placed one centimetre from an equal pole, the force between them is one dyne.** On measuring m_1 and m_2 in terms of these unit poles, the expression above becomes—

$$F = \frac{m_1 m_2}{d^2} \text{ dynes}$$

For further discussion see Chapter XVII.

As an example, find the force between a N pole of strength 250 units and a S pole of strength 80 units at a distance of 20 centimetres apart.

$$F = \frac{250 \times 80}{20^2}$$
$$= \frac{20000}{400}$$
$$= 50 \text{ dynes}$$

Also, the force is an attraction because the poles are of opposite kinds.

Magnetic field.—It must be remembered that although these magnetic considerations may appear to be of rather an abstract nature, yet they are necessary for a proper understanding of the measurement of electric current. The next step in our advance is the recognition of the fact that whenever a magnetic pole experiences a force, there must be magnets or electric currents in its neighbourhood, and although we may not have any exact knowledge of these magnets or currents, we may still recognise their effect upon the pole. It is then said that the pole is situated in a **magnetic field.** If we choose our pole to be a unit N pole, the direction of the force upon it is the direction of the field, and the value of the force acting upon it is the **strength of the magnetic field,** also sometimes called the **magnetic intensity** or the **magnetic force.**

Magnetic lines of force.—One of the most helpful devices in studying magnetic and electric fields is the idea of lines of force, due to Faraday. If we imagine a continuous line drawn so that its direction is always along the magnetic field, such a line is called a **magnetic line of force.** Further, if we can imagine a single unattached N pole, such a pole when placed in the magnetic field would travel along a line of force. Such a free pole cannot be obtained, as magnets always possess a S as well as a N pole. But the force on the S pole is always opposite in direction to that on a N pole, so that a small magnet freely suspended in a magnetic field will experience two opposite forces, one on its N pole and the other on its S pole, both acting along the line of force, but in opposite directions. The magnet will therefore set along a line of force.

The magnetic lines of force surrounding any magnet, or system of magnets, may therefore be found by means of a small compass needle, or suspended magnet, by placing it in successive positions in the magnetic field. In Fig. 7 the lines of force near a bar magnet have been drawn in this way. The student should perform the experiment for himself, starting at a point upon the magnet and marking the distant pole of the compass. Then move the compass, placing the other pole on this point and again marking the position of the distant pole, and so proceeding along the line of force. The whole space round the magnet can be mapped out in this way. The student should then treat a horseshoe magnet and then a pair of magnets in the same way.

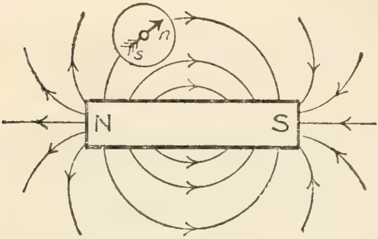


FIG. 7.—Determination of the magnetic lines of force due to a bar magnet by means of a compass.

Another way of exhibiting the lines of force, not so accurate as the above, is to place a piece of paper or a sheet of glass over the magnet and to sprinkle iron filings upon it. On tapping the sheet, the filings will arrange themselves in chains along the lines of force.

It must be noticed that lines of force always form continuous curves, arising upon a N pole and ending upon a S pole, and that no two lines of force meet or cross each other. In the case of an electric current, the first of these statements must be modified, but the second is rigidly true.

Strength of magnetic field.—On p. 11, the force which a unit N pole experiences is called the strength of the magnetic field. In some cases, the strength of magnetic field at any place can be calculated from a knowledge of the positions and strengths of any neighbouring poles. For example, let us find the strength of magnetic field at a distance of 15 centimetres from a magnetic N pole of strength 500.

Imagine m_2 to be a unit N pole placed at the point Then from the equation, $F=\frac{m_1 m_2}{d^2}$, we have—

$$\text{Force}=\frac{500\times 1}{15^2}=2\cdot 22 \text{ dynes}$$

for the force on the unit N pole. Thus the strength of magnetic field is 2·22 centimetre-gramme-second, or C.G.S., units.

The unit of magnetic field is sometimes called the **gauss,** after C. F. Gauss, the physicist, who did so much work in the development of the mathematical side of the study of magnetism. Thus we may say that in the above problem the strength of field is 2·22 gauss. The name is not in common use, but the student should be able to recognise its meaning in any case. Before being able to measure the strength of a magnetic field experimentally, a further consideration of the effect of a magnetic field upon a magnet must be made.

Magnetic moment.—Every magnet situated in a magnetic

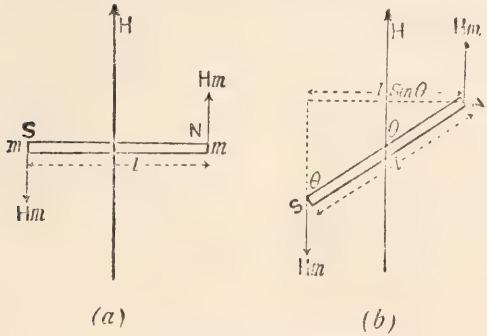


FIG. 8.—Couple acting on a magnet.

field experiences two forces, equal and opposite to each other. In Fig. 8 (*a*) let the magnet NS have poles of strength *m*, the magnet being situated in a field of strength H. The force on unit pole would therefore be H dynes, and on pole of strength *m* it will be H*m* dynes. The force on each pole is therefore H*m* and the two forces form a couple, the turning moment of which is H*ml*, where *l* is the distance between the poles, and therefore the perpendicular distances between the forces when the magnet lies at right angles to the field. The quantity *ml* or (strength of pole × distance between poles) is called the **magnetic moment** (M) of the magnet.

$$\therefore \text{ Turning moment of couple} = \text{M} \times \text{H}$$

when the magnet is at right angles to the field.

If the magnet is inclined at angle θ to the field, as in Fig. 8 (*b*), the perpendicular distance between the forces mH on the two poles of the magnet is $l \sin \theta$,

$$\therefore \text{ Turning moment of couple} = m\text{H} \times l \sin \theta$$
$$= ml\text{H} \sin \theta$$
$$= \text{MH} \sin \theta$$

The couple always tends to turn the magnet into the direction of the magnetic field, and if the magnet is free to turn it will do so, so long as the couple is not zero in value. For the couple to be zero, $\sin \theta = 0$, $\therefore \theta = 0$, and the magnet is then in the direction of the field.

Field due to a magnet.—There are two positions near a bar magnet, for which the strength of field due to the magnet may be calculated without difficulty.

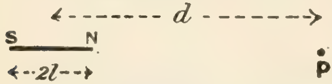


FIG. 9.—Strength of field due to bar magnet.

(i) Let the point P (Fig. 9) at which the field due to the magnet is to be calculated be situated in the axis of the magnet NS, that is, in line with N and S Imagine a unit N pole to be placed at P. Then its distance from N is $(d-l)$ and its distance from S is $(d+l)$. If then the pole strength of the magnet is m—

$$\text{Force on unit pole at P due N} = \frac{m}{(d-l)^2} \text{ dynes repulsion}$$

$$\text{,, \quad ,, \quad ,, \quad ,, \quad S} = \frac{m}{(d+l)^2} \text{ dynes attraction}$$

These forces are opposite and their resultant is therefore the difference between them.

$$\text{That is, strength of magnetic field at P} = \frac{m}{(d-l)^2} - \frac{m}{(d+l)^2}$$
$$= \frac{4mld}{(d^2-l^2)^2}$$

But the magnetic moment (M) of the magnet is $2ml$ since the length of the magnet is $2l$

$$\therefore \text{ Strength of magnetic field} = \frac{2\text{M}d}{(d^2-l^2)^2}$$

In many cases met with in practice, the magnet N is so small that l^2 is of no consequence in comparison with d^2, so that in this case, neglecting l^2 in the denominator, we have—

Strength of field on axis of magnet $=\dfrac{2M}{d^3}$ **C.G.S. units**

(ii) If the point P is situated on the line bisecting the magnet at right angles as in Fig. 10, the procedure, as before, is to imagine a unit N pole to be placed at P and to calculate the force on it due to N and to S and then find the resultant. These two forces are represented by PA and PB, and their resultant by the parallelogram of forces is then PR.

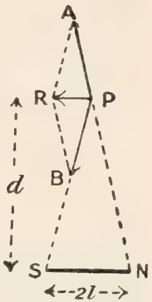


FIG. 10.—Strength of field due to bar magnet.

Now $PN^2=d^2+l^2$ and—

Force due to N or S on unit pole at $P=\dfrac{m}{PN^2}=\dfrac{m}{d^2+l^2}$

Also, by similar triangles,

$$\frac{PR}{PA}=\frac{NS}{PN}$$

and, $$NS=2l,\ PN=(d^2+l^2)^{\frac{1}{2}}$$

$$\therefore\ PR=\frac{m}{(d^2+l^2)}\cdot\frac{2l}{(d^2+l^2)^{\frac{1}{2}}}$$

$$=\frac{2ml}{(d^2+l^2)^{\frac{3}{2}}}$$

Remembering that $2ml=M$, the magnetic moment of the magnet,

$$\text{Strength of magnetic field}=\frac{M}{(d^2+l^2)^{\frac{3}{2}}}$$

As before, if l^2 is small enough to be neglected in comparison with d^2, this becomes M/d^3.

Strength of field at a point on a line bisecting the magnet at right angles $=\dfrac{M}{d^3}$ **C.G.S. units.**

The magnetometer.—We are now in a position to make

measurements of magnetic moments. For this purpose it is necessary to have some form of suspended magnetic needle, whose position upon a circular scale can be observed. An ordinary magnetic compass will do for this purpose, but it is usual to use a larger scale than that of the ordinary

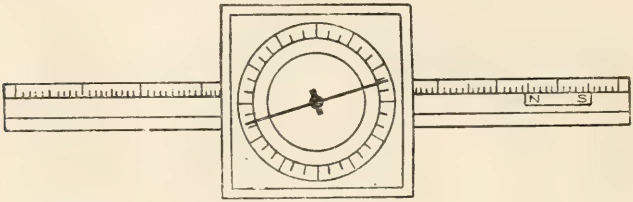


FIG. 11.—Magnetometer.

compass, and to provide also a horizontal graduated bar for fixing the position of the magnet to be experimented upon. Such an arrangement is called a **magnetometer.**

There are many forms of the instrument and a common type is shown in Fig. 11. The small magnetised needle at the centre of the circular scale is pivoted on a needle-point, and is provided with a long, light pointer which enables its position with respect to the circular scale to be observed. The bar magnet NS produces a magnetic field of strength $2M/d^3$ at the pivoted needle, and the distance d can be measured at once upon the straight scale.

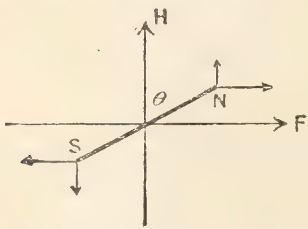


FIG. 12.—Forces on magnetometer needle.

In Fig. 12, the pivoted needle NS is shown of exaggerated size. If the bar magnet be absent the earth's magnetic field H will make the pivoted needle point along the direction of H. But the presence of the bar magnet will produce a field of strength F $\left(=\frac{2M}{d^3}\right)$ at right angles to H. The needle is therefore deflected and will come to rest when the couple due to H tending to rotate it back towards its original position is equal to the couple due to F rotating it away from this position. If m is the magnetic moment of the pivoted needle, the former couple is $Hm \sin\theta$ and the latter couple

$Fm \cos \theta$. The needle comes to rest when these couples are equal,

then, $$Fm \cos \theta = Hm \sin \theta$$

or, $$\frac{F}{H} = \frac{\sin \theta}{\cos \theta} = \tan \theta$$

But, $$F = \frac{2M}{d^3}$$

$$\therefore \quad \frac{2M}{d^3 H} = \tan \theta$$

or, $$\mathbf{\frac{M}{H} = \frac{d^3}{2} \tan \theta}$$

If the magnetometer were rotated through 90° and the bar magnet placed broadside to the needle as in Fig. 10, the field F due to the bar magnet is M/d^3, and in this case, $\mathbf{\frac{M}{H} = d^3 \tan \theta}$.

In making the observation of the deflection θ, it is necessary to read both ends of the pointer, because the pivot may not be accurately at the centre of the scale. Then the bar magnet must be reversed pole for pole by turning it round, and the observations repeated, because the magnetic poles of the bar may not be exactly symmetrical with respect to the middle of the magnet. Again, the readings must be all repeated with the bar magnet moved to the same distance on the other side of the suspended needle, because the middle of the straight scale may not be exactly at the point of suspension. Thus there are eight readings to be made in order to find the value of the deflection θ. The mean of the eight readings is free from the errors mentioned.

In this way it is possible to compare the magnetic moments of two magnets by placing them at the same distance d from the needle and measuring θ. Thus—

$$\frac{M_1}{M_2} = \frac{\tan \theta_1}{\tan \theta_2}$$

or the strength of magnetic field H at two places can be compared by finding the deflections produced by the same bar magnet at the same distance from the needle, with the magnetometer situated at the two places in turn. Thus—

$$\frac{H_1}{H_2} = \frac{\tan \theta_2}{\tan \theta_1}$$

Vibration of a suspended magnet.—The observations made with the magnetometer enable the comparison of magnetic moments and magnetic fields, but will not give the

absolute value of either. A further experiment is necessary, and this is supplied by the measurement of the time of vibration of a suspended magnet in a magnetic field. It will have been noticed that a suspended magnet when disturbed from its position of equilibrium takes some time to settle down to rest again. The time of one of the vibrations made in settling down to rest depends upon the strength of the magnetic field, the magnetic moment of the magnet, and upon its mechanical properties. The relation between these quantities will not be proved here; it is—

$$T=2\pi\sqrt{\frac{I}{MH}}$$

where T is the time in seconds for a complete vibration, and M and H have the same meanings as before. The quantity I is called the moment of inertia of the magnet and for a bar magnet is,—mass $\times\left(\frac{\text{length}^2+\text{breadth}^2}{12}\right)$.

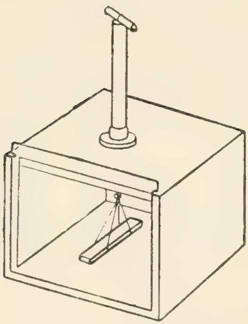


FIG. 13.—Vibration box.

The bar magnet is suspended by a fine thread inside a box as shown in Fig. 13. Two sides of the box are of plain glass, to enable the vibrations to be observed. The vibration should only be a few degrees on either side of the rest position. Twenty or fifty complete vibrations may be timed by means of a stop-watch and the time T of one vibration calculated.

Then,
$$T=2\pi\sqrt{\frac{I}{MH}}$$

or,
$$MH=\frac{4\pi^2 I}{T^2}$$

Absolute determination of strength of magnetic field.—Provided that the bar magnet used in the vibration experiment is the same one that is used for the magnetometer experiment (p. 17), and that the place of vibration is the same as was originally occupied by the magnetometer needle,

M and H have the same values in both experiments. On taking the product of the values obtained for M/H and for MH, we have—

$$\frac{M}{H} \times MH = M^2$$

and so the value of the magnetic moment of the bar magnet is known in absolute C.G.S. units.

Similarly on finding the quotient, as follows:

$$\frac{MH}{\frac{M}{H}} = H^2$$

the absolute value of H is found. This value is in absolute C.G.S. units or gauss.

Comparison of fields by vibration methods.—It frequently happens that the ratio of two field strengths is required, and not the absolute value of either field. In this case the same magnet is suspended in the two magnetic fields in turn and its time of vibration found. Then for the first field, H_1, we have—

$$MH_1 = \frac{4\pi^2 I}{T_1^2}$$

and for the second field—

$$MH_2 = \frac{4\pi^2 I}{T_2^2}$$

Dividing one of these expressions by the other we have—

$$\frac{H_1}{H_2} = \frac{T_2^2}{T_1^2}$$

since M and I are the same in both cases. If the strength of one of the fields is known, that of the other can now be calculated from the times of vibration of the magnet in the two fields.

It is sometimes more convenient to take the ratio of the number of vibrations made in the same time (one minute, or five minutes) in the two fields, rather than the ratio of the times of vibration. Remembering that the number of vibrations made in a given time varies inversely as the time of vibration, it follows that—

$$\frac{H_1}{H_2} = \frac{n_1^2}{n_2^2}$$

where n_1 and n_2 are the numbers of vibrations made in the same time in the fields H_1 and H_2.

By this means the student may find the relative strengths of field at various points in the neighbourhood of a bar magnet.

Field strength and lines of force.—Any one plotting carefully the lines of force around a bar magnet must be struck by the fact that in those places where the magnetic field is strong, the lines of force are crowded together ; where the field is weak the lines are separated by greater distances. This consideration leads to the idea that the magnetic field may be represented in strength as well as in direction by the lines of force. This may be shown mathematically to be the case, and it follows that by choosing the lines properly, their use may be considerably extended. It is possible to imagine lines of force in the magnetic field to be distributed in such a way that their number per square centimetre is numerically equal to the strength of field at each part of the field, provided that the square centimetre is taken at right angles to the direction of the field.

The field strength is then spoken of in terms of lines per square centimetre, a field of unit strength having one line per square centimetre. In the case of a uniform magnetic field the lines of force are parallel and equidistant, and the number passing through any area is, (strength of field) $\times$ area $= H \times A$.

Magnetic induction.—It was seen on p. 12 that the magnetic lines of force due to a magnet appear to arise upon the N pole, and end upon the S pole. It is not only possible, but reasonable, to suppose that the lines meeting the magnet at the S pole do not cease to exist, and come into existence at the N pole, but that they pass through the steel, forming, with the external lines, complete paths as in Fig. 14. Of course the lines cannot be traced into the steel by means of an exploring compass needle as they can be traced in the surrounding space, but there are means of determining their number in the steel itself.

Here a difficulty arises, because within iron or steel the number per square centimetre does not represent the strength of magnetic field or force which would be exerted on a unit magnetic pole. For this reason the lines are not called lines of force when they lie within a magnetic material. They are called **magnetic lines of induction,** and their number per square centimetre is the value of the **magnetic induction** within the iron, the symbol for which is B.

It is generally considered that magnetic lines of induction are always closed curves. They may lie partly in iron and partly in air, or entirely in iron, or entirely in air. But when they lie in air their number per square centimetre is the strength of magnetic field, and they are then also called lines of force. Thus in air or any other non-magnetic material B = H, but this relation is not true for iron, steel, or any other magnetic material. For the three metals iron, nickel and cobalt, B is much greater than H, which means that the force upon a unit pole situated within the metal is much less than the number of lines of induction per square centimetre. If B is the value of the magnetic induction within the metal, due to the metal being situated in a magnetic field of strength H, the ratio

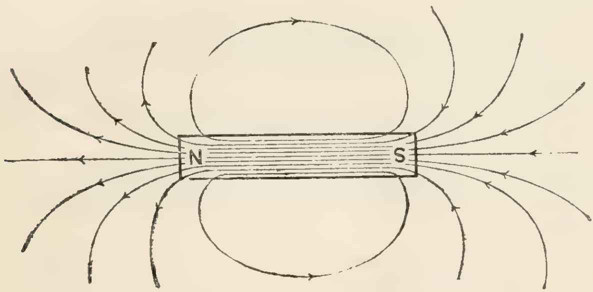


FIG. 14.—Lines of force and lines of induction.

B/H is the **magnetic permeability** of the medium and is usually designated by the letter μ.

Thus, $$\mathbf{B}=\mu\mathbf{H}, \text{ or } \frac{\mathbf{B}}{\mathbf{H}}=\mu$$

The relation between these magnetic quantities will be treated more fully in Chapter XVII, but it may be noted here that the value of μ for iron, nickel and cobalt, at ordinary magnetic fields, may be anything between 3000 and 500. These three metals form a group by themselves and are called the **ferro-magnetic** metals.

EXERCISES ON CHAPTER II

1. Describe simple experiments which illustrate the differences in the magnetic properties of soft iron and hard steel.

2. State qualitatively the laws of force between magnetic poles, and describe experiments for proving them.

3. Describe what happens when a hard steel magnet is broken into halves, and show how this experiment bears upon the molecular theory of magnetisation.

4. What is the quantitative law of force between two magnetic poles? Find the force between a N magnetic pole of strength 75 and a S magnetic pole of strength 45, placed 15 cm. apart.

5. Two magnets, one having pole strength 36 and length 8 cm., and the other pole strength 60 and length 12 cm., are placed in line with each other. Find the force between the magnets if their middle points are 15 cm. apart.

6. What is the magnetic moment of a magnet? Find the couple acting on a magnet whose magnetic moment is 260 C.G.S. units when situated (*a*) perpendicular to, and (*b*) at an angle of 60° to, a magnetic field of strength 3·5 C.G.S. units.

7. Calculate, from first principles, the strength of field at a point on the axis of a magnet and 50 cm. from the middle point of the magnet, whose pole strength is 80 and length 10 cm., and also by using the expression $2M/d^3$.

8. Calculate the strength of field at a point at a distance of 40 cm. from the middle of a magnet, of moment 250 and length 10 cm., the point being equidistant from the two poles. Use both the exact and the approximate expressions.

9. Describe the magnetometer, and explain its use for comparing the strengths of magnetic field at two places.

10. From the following data, calculate the strength of the earth's magnetic field as measured by means of the magnetometer.

Distance of middle of magnet from needle, in end-on position=30 cm., deflection=10°. Moment of inertia of magnet=106. Time of vibration=8·3 sec.

11. If a suspended magnet makes 15 vibrations in 40 seconds at a place where the earth's field is 0·18 C.G.S. units, and 30 vibrations in 50 seconds at another place, find the strength of the earth's magnetic field at the latter place.

12. A magnet whose magnetic moment is 180 C.G.S. units makes 16 vibrations in 90 seconds when suspended at a place where the strength of magnetic field is 0·22 gauss. Find the moment of inertia of the magnet.

13. A small suspended needle makes 25 vibrations per minute when in the earth's field alone, and 15 vibrations per minute when a magnet is brought near it, the needle being 16 cm. due west of the magnet, whose N pole points north. Find the magnetic moment of the magnet if the strength of the earth's field is 0·18 gauss.

14. A suspended needle (N end points north) is situated 30 cm. north of a bar magnet whose S end points north, and the needle makes 8 vibrations per minute. The bar magnet is now reversed, end for end. How many vibrations per minute will the needle now make, if the magnetic moment of the magnet is 1485 C.G.S. units, and the strength of the earth's field is 0·22 gauss.

15. Show that the strength of field at a point on the axis of a magnet is equal to $2M/d^3$ when the distance from the magnet is great compared with the length of the magnet.

16. A point A is situated on the axis of a bar magnet and B on a line bisecting the magnet at right angles. If the strengths of magnetic field at A and B are equal, show that the ratio of A's distance to B's distance from the middle of the magnet is $\sqrt[3]{2}$.

CHAPTER III

MAGNETIC FIELDS AND ELECTRIC CURRENTS

Magnetic field due to electric current.—It now becomes necessary to apply the knowledge of magnetic fields gained in the last chapter to the application and measurement of electric currents. From the experiment on magnetising a piece of iron rod (p. 4) by means of an electric current, it is seen that the current produces a magnetic field. If the current flows in a wire bent into a helical form called a **solenoid,** as shown in Fig. 15, the magnetic field produced

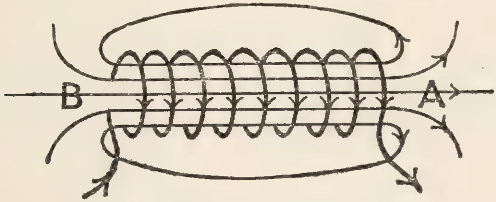


FIG. 15.—Magnetic field due to a solenoid.

can be investigated by means of a small compass needle, and will be found to be of the form shown in the diagram. Magnetic lines of force emerge from one end of the solenoid, A, pass round externally, and enter at the other end B. A comparison with Fig. 14 shows that the end A of the solenoid in Fig. 15 behaves like the N pole of a magnet, and the end B like a S pole.

If a straight wire is used for the current instead of a solenoid, the magnetic field may again be investigated by means of a compass needle, by passing the wire through a horizontal piece of paper or cardboard and tracing the magnetic lines of

force in the manner described on p. 12. The lines will then be found to be circles having their centres upon the wire. Thus in Fig. 16 the magnetic field at C will be in the direction CD, at right angles to the wire and to the perpendicular CE from the point on to the wire. The magnetic line of force passing through C is therefore a circle, and a compass needle at NS will set tangentially to this circle.

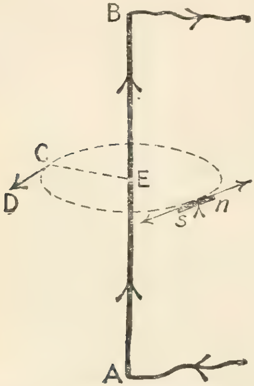


FIG. 16.—Magnetic field due to a straight wire in which an electric current is flowing.

Direction of current.—It is clear that a current in a wire must have one particular direction along the wire, but up to the present we have said nothing about this direction. In the early history of the study of electricity there was nothing to decide upon one direction rather than its opposite for the positive flow of the current. Thus the current might be flowing from A to B, or from B to A in the wire in Fig. 16. The convention became gradually adopted, of considering the current to flow from A to B, and this conventional rule may be described in several ways. According to Ampère if we imagine ourselves to be swimming in the wire in the positive direction of the current and to look towards such a point as C, then a N magnetic pole situated at C would experience a force directed towards the **left hand.** Or, if a right-handed screw be rotated so that it would travel in the direction of the current, the direction of rotation of the screw is the direction of the magnetic lines of force.

But perhaps the most useful rule is that of considering the wire to be grasped in the **right hand** with the thumb pointing along the wire in the positive direction of the current, then the fingers indicate the direction of the magnetic lines of force.

The above convention for determining the positive direction of the current is still retained although, had we to build up our system while possessing our present knowledge of the nature of current, it is probable that the opposite direction would be chosen.

The reason for this is that the current is now known to consist of the motion of certain bodies in the opposite direction to that previously chosen as the conventional positive direction of the current, without the travel of anything in the positive direction. The difficulty is got over by considering the bodies constituting the current to consist of negative electricity. These bodies are called **electrons** and will be explained further in Chapter XIII.

Unit of electric current.—Our knowledge of the intimate connection between the magnetic field produced by an electric current and the current itself is due to Andrée M. Ampère (1823), and, following his suggestions, the strength of an electric current is defined from the magnetic field which accompanies it. For this purpose, the simplest form of the conducting circuit is that of a circle of wire in which the current flows, as in Fig. 17. At the centre, the magnetic field is perpendicular to the plane of the circle, as may be found on plotting the field by placing a horizontal sheet of paper to pass through the centre of the coil, and using a compass needle.

FIG. 17.—Magnetic field of circular circuit.

If the circle has a radius of one centimetre, then the current flowing along an arc of the circle one centimetre long which would produce a magnetic field of unit strength at the centre of the circle is taken as the C.G.S. unit of current.

For any other length of arc, say l centimetres, the magnetic field would be l times as great, and for a radius of circle other than unity, say r centimetres, the strength of magnetic field at the centre for any given current may be shown experimentally to vary inversely as r^2. Hence, for a complete circle of one turn the magnetic field at the centre, for unit current, will be l/r^2. But $l=2\pi r$, therefore field strength will be $\frac{2\pi r}{r^2}=\frac{2\pi}{r}$. For a current of strength i units, the strength of field will be $2\pi i/r$, and if there be n turns instead of one the field will be n times as great.

$$\therefore \left.\begin{array}{l}\text{Strength of magnetic field}\\ \text{at centre of circular coil}\end{array}\right\}=\frac{2\pi n i}{r} \text{ C.G.S. units}$$

Measurement of electric current.—Since the strength of an electric current is related to the magnetic field it produces, as described above, and we have studied in the last chapter how a magnetic field may be measured, we are now in a position to apply our knowledge to the measurement of current. In the equation for the deflection of the magnetometer needle $\left(\frac{F}{H}=\tan\theta\right)$ given on p. 17, the magnetic field F is due to a bar magnet. But if the suspended needle is situated at the centre of a circular coil in which an electric current is flowing, the magnetic field F will be due to the current. Now,

$$F=\frac{2\pi ni}{r}$$

$$\therefore \frac{2\pi ni}{rH}=\tan\theta$$

If n, r and H are known and θ be observed, the current i is determined in absolute C.G.S. units.

Tangent galvanometer.—The apparatus for carrying out the measurement of electric current by making use of the above relation is known as the **tangent galvanometer.** Taking a horizontal section through the circular coil, we see the arrangement of coil and suspended needle in Fig. 18, in which the size of the suspended needle is greatly exaggerated for the sake of clearness. The plane of the circular coil must be parallel to the direction of the earth's magnetic field H, that is, it must lie in the **magnetic meridian.** Then the field H produces a couple tending to turn the needle into the magnetic meridian. The magnetic field F due to the current in the coil is at right angles to the plane of the coil, and produces a couple which tends to make the needle set at right angles to the magnetic meridian. The needle comes to rest at angle θ to the meridian, when the resultant of F and H is in the direction of the needle, that is, when $\frac{F}{H}=\tan\theta$, or, $\frac{2\pi ni}{rH}=\tan\theta$;

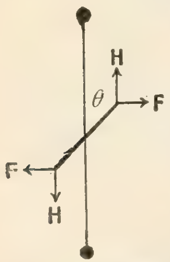


FIG. 18 —Forces on magnet of tangent galvanometer.

$$\therefore i=\frac{rH}{2\pi n}\tan\theta$$

A common type of tangent galvanometer is shown in Fig. 19. The circular coil is wound in a groove in a circular frame, the needle being suspended at its centre by means of a fine silk fibre. A pointer at right angles to the needle, and a circular scale for observing the deflection, are provided as in the magnetometer (p. 16). In fact, the tangent galvanometer is a form of magnetometer in which the deflecting magnet is replaced by the circular coil of wire carrying an electric current. Since the needle necessarily lies above the scale, an error of reading might be made if the eye is not situated vertically over the needle at the moment of observing θ. For this reason a plane mirror occupies the space inside the circular scale, and the eye is so placed over the

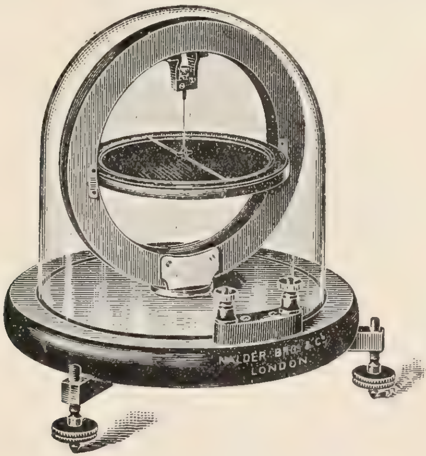


FIG. 19.—Tangent galvanometer.

needle that the optical image of the needle in the mirror appears to coincide with the needle itself, at the moment that the deflection is read. This ensures the eye being situated vertically over the needle.

The number of turns in the coil (n) and the radius of the turns (r) can be measured before the current observations are made. If absolute measurements of the current are required, the value of the earth's field (H) must be determined by a magnetometer method (p. 19) before the current experiment. In measuring θ, both ends of the pointer must be observed. Then the current must be reversed, and the readings repeated, and the mean of the four readings used for calculating the current. If the readings with

current reversed differ by more than half a degree from the first readings, it means that the pointer is not at right angles to the magnetic needle, and in setting the instrument with the pointer at the zero of the scale, the plane of the coil is not parallel to the magnetic meridian. It is necessary, then, to twist the needle slightly with respect to the pointer and to repeat the observation. This must be done several times, until the readings for the two directions of the current do not differ by more than half a degree. When this is attained, any further correction is performed by taking the mean of the four observations of θ, as described above.

It may be noticed that if the current can be measured by other means (pp. 88 and 117) the tangent galvanometer becomes a very convenient instrument for measuring H, the strength of the earth's magnetic field.

Throughout these measurements it is desirable to take care that the deflections shall be of convenient size. If the deflection is too small, then the error introduced in its reading may be a large fraction of the deflection itself, so that the percentage error of observation is high. On the other hand, if the deflection is very large, a large variation in current will only make a small increase in the deflection, since the tangent of an angle increases rapidly with increase in the angle for high values. Thus an error of half a degree in the observation will correspond to a large error in the value of the current. The most efficient value of θ is about 45°, and in practice θ should never be less than 15° or greater than 65°.

Comparison of currents.—It frequently happens that the absolute value of the current is not required, but merely the ratio of the values of two currents. It is not then necessary to measure n, r, or H, their values being constant. In this case the four values of θ for each current are observed and the mean values, θ_1 for current i_1, and θ_2 for current i_2, are found.

Then, $$i_1=\frac{r\mathrm{H}}{2\pi n}\tan\theta_1,\text{ and, } i_2=\frac{r\mathrm{H}}{2\pi n}\tan\theta_2$$

and, $$\frac{i_1}{i_2}=\frac{\tan\theta_1}{\tan\theta_2}$$

If one of the currents is known, the other can then be calculated.

Sensitive galvanometer.—Owing to its form, the tangent galvanometer is adapted to the measurement of large currents, and it is necessary to modify its design if very small

currents are to be measured. This modification may be effected partially by increasing the number of turns in the coil, and by diminishing their radius, although the latter alteration destroys the tangent law, since the needle is no longer very small with respect to the coil. Since sensitive galvanometers are not used for the absolute determination of current, the loss of the tangent relationship is of no consequence.

A simple and useful form of galvanometer, in which these principles are employed, is illustrated in Fig. 20. The coil A

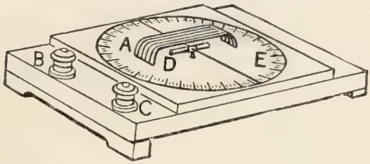


FIG. 20.—Simple galvanometer.

consists of thirty or forty turns of No. 26 silk-covered copper wire, and its ends are soldered to the terminals B and C. The needle D is provided with an agate cup which rides on a fixed needle-point, while the light pointer, attached at right angles to the needle, moves over the circular scale E.

The next step in increasing the sensitiveness consists in employing a magnet to diminish the controlling field H. Such a magnet is placed, with its N pole pointing north, near the galvanometer, and is moved nearer to increase the sensitiveness, and further away to decrease it. It is called a **controlling magnet,** and has the further advantage of providing a simple means of bringing the needle to its zero position so that the pointer rests over the zero of the circular scale.

Another method of increasing the sensitiveness of the galvanometer is to decrease the controlling effect of the earth's field H, without altering the actual field, by using two needles rigidly attached together, but with unlike poles at each end of the couple, as in Fig. 21. Such an arrangement is called an **astatic couple.** The effects of the controlling magnetic field on the two needles are opposite; therefore the resultant control is much feebler than if only one needle were used. The needles are never exactly

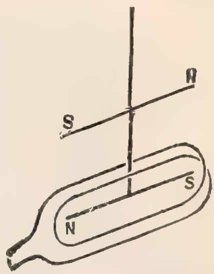


FIG. 21.—Astatic couple.

equal in magnetic moment, so that there is always some control remaining, as is necessary. Since the coil surrounds only one of the needles it produces its full deflecting effect.

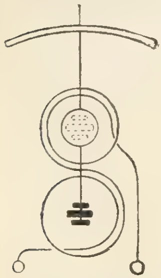


FIG. 22.—Astatic reflecting galvanometer.

In the arrangement shown in Fig. 22, due to Lord Kelvin, two coils are used, one surrounding each magnet. Since the coils are oppositely wound, and the needles are opposite in direction, the deflecting couples are **both in the same direction,** so that still greater sensitiveness is obtained. The form of the Kelvin astatic galvanometer is seen in Fig. 23, in which the pair of coils is seen, as well as the controlling magnet, pivoted upon a pillar, up or down which it can be made to slide.

Another type of sensitive galvanometer will be described in Chapter IX.

Reflecting galvanometer.—Having exhausted the means of increasing the real sensitiveness of the galvanometer, we must now turn our attention to an improvement in the method of observing the deflection. In both the magnetometer (p. 16) and the galvanometer (p. 27) we have considered the deflection θ to be measured by means of a horizontal pointer attached to the needle and moving over a circular scale. With this arrangement, accuracy of reading can never be attained, and small deflections can scarcely be observed Increasing the length of the pointer is out of the question if a mechanical pointer is used, but if the pointer is a beam of light reflected by a mirror attached to the needle, a new range of sensibility is available.

FIG. 23.—Kelvin reflecting galvanometer.

To understand this, refer to Fig. 24. M is a small concave mirror, silvered on the back

surface, to which the magnet, consisting of two or three pieces of magnetised watchspring, is attached. The arrangement of magnet, mirror and suspending silk or quartz fibre is shown at M′. F is a filament, which is maintained in a state of incandescence by an electric current, the light from which is concentrated by the lens L on to the mirror M. On the front surface of L is a diamond scratch, and M, being concave, produces an optical image of this scratch, upon the horizontal scale S. The scale S is usually semi-transparent, so that the image can be seen from the side remote from the mirror. As the magnet is deflected, the mirror rotates, and the image travels along the horizontal scale, upon which its position can be read. The distance from the scale to the mirror is generally about 1 metre, so that a very small rotation of the mirror gives a fairly large travel of the spot upon the scale. There is a further advantage in this method, in that the reflected beam of light moves through

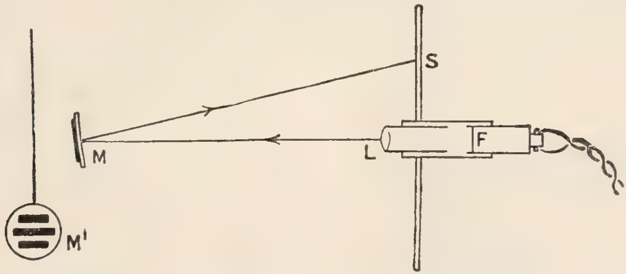


FIG. 24.—Optical system of reflecting galvanometer.

twice the angle through which the mirror turns. The only disadvantage of this method is that the room in which the experiment is made must be darkened. When this darkening is objectionable the method may be modified by using a telescope and scale instead of a lamp and scale. This arrangement is seen in Fig. 25, where the arrangement is shown for a magnetometer, although it applies equally well to the galvanometer. In this case a plane mirror is used, which, of course, produces an image of the scale, as far behind the mirror as the scale is in front. The telescope is focussed upon this image in the mirror, and as the mirror rotates, the image of the scale appears to move across the field of vision as seen through the telescope. A vertical cross wire in the eyepiece of the telescope enables the observer to read the deflection.

Detector.—For many practical purposes, a simple form of galvanometer is employed, which merely indicates when a current is passing without making any measurement of the strength of

current. Such an instrument is called a detector, an example of which is shown in Fig. 26. The magnet NS is pivoted so that it can rotate in a vertical plane. A pointer P is attached to the magnet

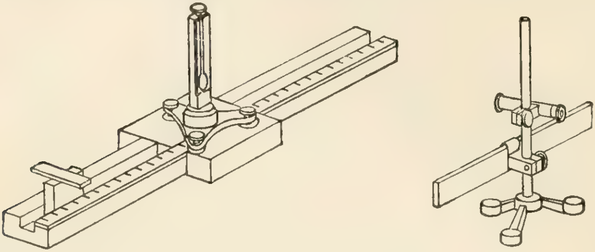


FIG. 25.—Reflecting magnetometer.

so that it rotates with it. The pointer is weighted so that it and the magnet set vertically when no current is passing in the coils. A current in one direction causes the N pole to be deflected to the right, and a current in the opposite direction causes the N pole to be deflected to the left. The detector is useful for detecting a break-down in insulation of a wire or cable and for identifying the two ends of a wire, part of which is hidden. It is also a useful receiver for telegraphic signals, when the stops A and B may be replaced by two gongs of different pitches so that current in either direction may be recognised by the sound made when the pointer P strikes one gong or the other, the different pitches corresponding to the dots and dashes of the Morse code. Such a receiver is commonly employed in railway telegraphy. For submarine telegraphy a delicate galvanometer of the suspended coil type is used.

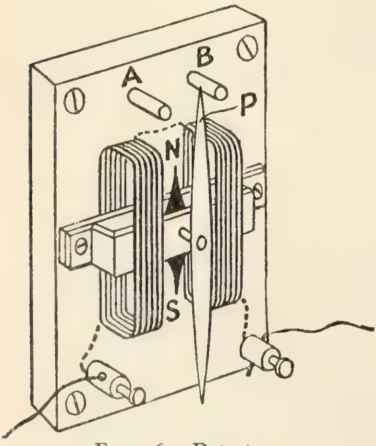


FIG. 26.—Detector.

Simple electric telegraph.—At a very early date, the magnetising effect of the electric current was used for

signalling purposes. The first forms of electric telegraph consisted of a battery B, a detector G, and a key K (Fig. 27), the cables L_1 and L_2 being used to complete the circuit and connect the two stations. One of the lines L_1 or L_2 may be dispensed with and the earth used as a return circuit, if the leading wires are connected to plates well embedded in the

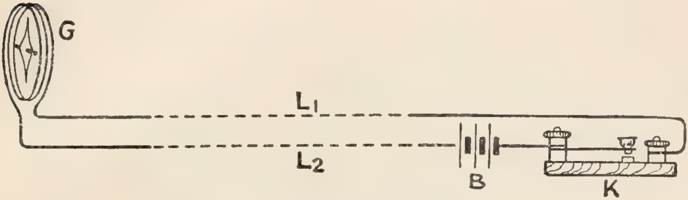


FIG. 27.—Simple electric telegraph.

ground, one at each station. On depressing the key K for a long or a short time the deflection of the galvanometer or detector G lasts for a long or a short time, corresponding to a dash or a dot of the Morse code.

Morse key.—The simple arrangement of Fig. 27 would have to be duplicated for sending messages in both directions. In

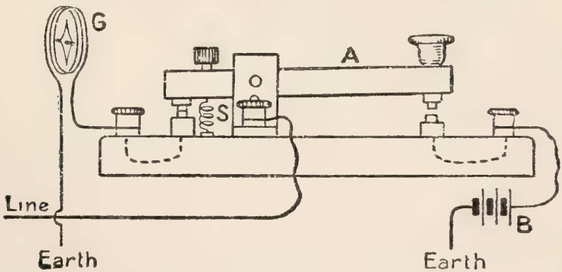


FIG. 28.—Morse key used for sending.

order to avoid this duplication the **Morse key** is employed, as shown in Fig. 28, one such arrangement being situated at each station, the two stations being connected by the line and earth to complete the electric circuit. On depressing the lever A, the detector G is insulated from the line and the battery B produces a current which is indicated on the detector at the distant station.

On releasing A the spring S pulls the arm back so that contact is made at the back stop and the detector G is connected to the line, ready to receive signals.

Exercises on Chapter III

1. Describe by means of sketches the form of the magnetic field due to a straight wire, a circular coil, and a solenoid, carrying an electric current.

2. Give the rule connecting the direction of an electric current with the direction of the magnetic field due to it, and describe an experiment by which you would establish this rule.

3. What is the unit of electric current? Deduce an expression for the strength of magnetic field at the centre of a circular coil in which a current is flowing.

4. Calculate the couple which would act on a small magnet whose magnetic moment is 18 C.G.S. units, when situated at the centre of a circular coil of 20 turns of radius 12 cm. when a current of 2·5 C.G.S. units is flowing, the magnet lying in the plane of the coil.

5. A small magnet makes 20 vibrations per minute in the earth's field alone, and 30 vibrations per minute when current flows in a circular coil of 4 turns of radius 25 cm., when the magnet is at the centre of the coil and the magnetic field of the current is added to the earth's field. If the strength of the earth's field is 0·2 gauss, what is the current in the coil?

6. A small magnet is situated at the centre of a circular coil of 30 turns of radius 15 cm. with the plane of the coil facing north and south. The magnet makes 50 vibrations per minute when the current is in one direction and 25 vibrations per minute with the current reversed. Calculate the strength of the earth's magnetic field if the current is 0·03 C.G.S. unit.

7. Calculate the current in a tangent galvanometer if the deflection is 60° when the number of turns is 10 and radius 15 cm., the strength of the earth's magnetic field being 0·18 gauss.

8. Describe the necessary form of a sensitive galvanometer and the reasons for the form described.

9. What is an astatic magnetic couple, and what is its use?

10. A current of 1·5 C.G.S. units produces a deflection of 50° on a tangent galvanometer. A second current produces a deflection of 35°; calculate its value.

11. Describe how you would make a galvanometer (*a*) more sensitive and (*b*) less sensitive by means of a controlling magnet.

12. Describe some form of reflecting magnetometer or galvanometer.

13. Give an account of some simple form of electric telegraph in which the Morse key is used.

14. If the same current passes through two tangent galvanometers where H has the same value, but in one having 30 turns of radius 14 cm. the deflection is 25°, what is the deflection in the other galvanometer which has 20 turns of radius 12 cm.?

CHAPTER IV

STATIC ELECTRICITY

Electricity at rest.—The term "electric current" implies that there is something moving along the conductor in which the current "flows," but it is not possible to obtain information on this question from the phenomenon of the heating or the magnetic field accompanying the current. We must turn our attention to the earliest known experiment bearing upon electricity, namely, the rubbing of amber with silk or wool. The amber is then capable of attracting light bodies to itself. From the Greek name for amber, "elektron," the word electricity is derived.

An effect similar to that produced by rubbing amber may be obtained from any non-conducting substance on rubbing it with a non-conductor, provided that there is fairly intimate contact between the two. For this reason one of the bodies is usually hard, such as glass, ebonite, sealing-wax, etc., and the other soft, such as fur, wool, silk, etc. The effect is in fact more general than the above statement implies, for any two substances may be used, conducting or non-conducting, although in the case of conductors, the rubbed body must be insulated, or the effect vanishes instantly.

When proper precautions are taken, the bodies not only attract light bodies to themselves, but attract or repel one another. They are then said to possess charges of **electricity.** The properties of electricity at rest upon the body differ in many respects from its properties when moving along a conductor and constituting an electric current, and we owe it to Michael Faraday that we recognise the identity of the electricity in the two cases. Faraday took great pains to show that the electricity produced by friction could, when passing along a wire, produce all the effects by which we recognise an electric current caused by a battery. On the other hand, it is comparatively easy to show that electricity at rest can be produced by a battery. Thus, while recognising the fundamental identity of the electricity as met with in the two cases, it is still convenient to study them separately for a time. Thus

static electricity and the electric current are not separated in nature, but the effects by which they are recognised give rise to two methods of study.

Early discoveries.—The experiment of rubbing amber may be repeated with ease, provided that the amber and the cloth are thoroughly dry. Small pieces of paper make useful light bodies to exhibit the attraction of the amber. The effect is still more easily exhibited if a rod of ebonite is used instead of the amber and a piece of well-dried fur, such as cat's skin, is used for the rubber.

It was found in very early times that there are two distinct kinds of electricity, one produced on the glassy substances, which was at first called *vitreous* electricity, and the other on the substances such as amber or resin, called *resinous* electricity. These names were abandoned for the terms **positive electricity** for the former, and **negative electricity** for the latter.

Forces between the two kinds of electricity.—If a rod of ebonite be rubbed with dry fur, the charge of electricity it acquires is of the negative kind. Let the rod be suspended by a silk fibre S as in Fig. 29. Then, on bringing another ebonite rod rubbed with fur, near the suspended rod, it will be seen that the suspended rod is repelled, whether the end A or B is approached. On bringing a glass rod rubbed with dry silk near the suspended ebonite rod there will be an attraction between the two. If AB be replaced by a glass rod rubbed with silk and therefore positively charged, and the ebonite rubbed with fur be brought near it, the ebonite will attract the glass. But glass rubbed with silk will repel it. Hence we may conclude that—

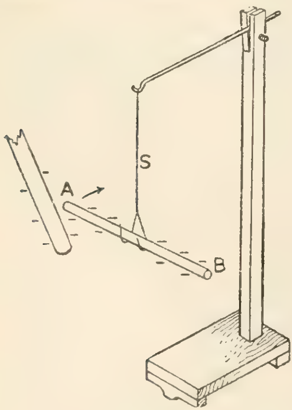


FIG. 29.—Experiment to illustrate force between charged bodies.

Like charges of electricity repel each other,
Unlike charges of electricity attract each other.

There is a similarity between this law and the law of force between magnetic poles. But it must be remembered that, whereas a piece of iron or steel always has two poles of opposite kinds, a body may be charged all over with one kind of electricity. It does not possess poles as in the case of a magnetised body.

Unit charge.—Following the analogy of magnetic poles (p. 10), it is possible to define unit quantity of electricity. Thus **on placing two equal charges one centimetre apart, if the force between them is one dyne, they are said to be unit charges.** In this definition the supposition is made, that the charges are situated in a vacuum, or, what is very nearly the same thing, in air. For other media the force will not be the same, but this question will be deferred until later (p. 60).

Law of force between charges of electricity.—It has been proved by Cavendish and Maxwell to a very high degree of accuracy, that the force between any two charges of electricity **varies inversely as the square of their distance apart,** provided that the charges are situated at points. It is not possible to prove this law with any reasonable accuracy by direct experiment, but our belief in it becomes firmly established when it is found that application of it always leads to results which are consistent with experimental results, while any other law would lead to results which are not borne out by experiment. The Cavendish and Maxwell experiments are of this nature and are consistent with the inverse square law only. It follows that if we measure two quantities of electricity q_1 and q_2 in C.G.S. units, the force between them when situated d centimetres apart is given by—

$$\text{Force} = \frac{q_1 q_2}{d^2} \text{ dynes.}$$

The unit of quantity of electricity defined above is derived entirely independently of the units of magnetic pole and current, and the units of quantity of electricity and those derived from them and used in the study of static electricity are called the **electrostatic units,** while those derived from the magnetic pole are called the **electromagnetic units.** We shall see later that there is an important relation between the electrostatic units and the electromagnetic units.

Electric field.—On placing a unit of positive electricity

at any point in the neighbourhood of other electric charges it experiences a force, and the point is said to be situated in an electric field. **The value of the force in dynes acting on the unit positive charge is the strength of the electric field, or electric intensity,** at the point. Also the direction of the force is the direction of the electric field.

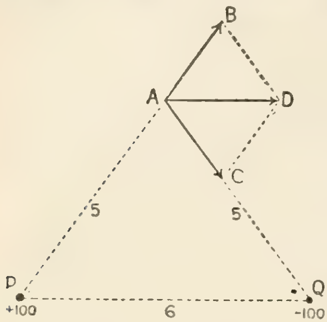


FIG. 30.—Problem.

As an example, let us find the strength of electric field at a point A (Fig. 30) at distances 5 cm. from two charges of $+100$ and -100 units respectively, placed at P and Q, 6 cm. apart.

Imagine a unit positive charge placed at A. Then, force on unit charge due to charge at $P=\frac{100\times 1}{5^2}=4$. This force of 4 dynes may be represented by AB. Similarly the force due to charge at $Q=\frac{100\times 1}{5^2}=4$ and may be represented by AC.

The resultant of the forces AB and AC is AD, and from similar triangles—

$$\frac{AD}{AB}=\frac{PQ}{AP}$$

$$\therefore \quad \frac{E}{4}=\frac{6}{5}$$

$$E=\frac{4\times 6}{5}=4{\cdot}8 \text{ C.G.S. units}$$

where E is the strength of the electric field.

Electric lines of force.—By drawing lines in the electric field in such a way that their direction is everywhere the direction of the field, we obtain curves which are called **electric lines of force.** In Fig. 31 (*a*) are seen the electric lines of force due to two equal charges of electricity of opposite signs, and in Fig. 31 (*b*) those due to two equal positive charges. The arrows show the positive direction

of the lines of force, that is, the direction in which a positive charge would be urged. It will be seen that electric lines of force arise on a positive charge and end upon a negative charge.

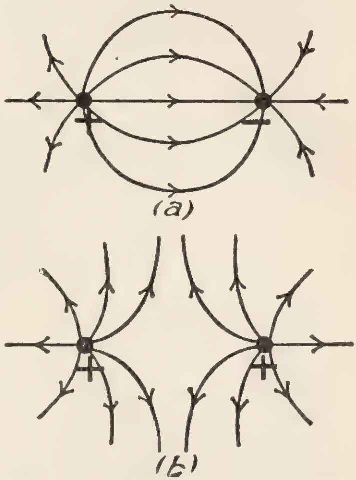


FIG. 31.—Electric lines of force due to two charges.

It must also be noticed that when the charge upon which the lines of force arise is situated upon a conductor, the lines of force where they spring from or end upon the conductor **must be at right angles to the conducting surface.** For if they are not, but are inclined to the surface, this means that the electric field is inclined to the surface. It may then be resolved into two components, one perpendicular to the surface and one parallel to it. The component of field parallel to the surface will cause a current in the surface. If, then, the electricity is at rest upon the conducting surface it follows that there is no component of the electric field parallel to the surface, that is **the electric field at the surface must always be at right angles to the surface.**

Just as in the case of magnetic fields, it is possible to draw the lines of force so that their number per square centimetre is equal to the strength of the field. The idea of lines of force we owe to Faraday, and it has been of the greatest importance in studying electrical phenomena. For example, if the lines of force are drawn in any given case, it is possible to deduce from them the forces which act upon the charges in the field, by remembering that the lines of force tend to shorten, or to behave as though they were lines of some material under tension. Thus, in Fig. 31 (*a*), the lines in tending to shorten would draw the two charges together. In Fig. 31 (*b*) the lines, which must end on negative charges somewhere, will, in shortening pull the two positive charges apart. These forces are of the kind we find in practice to be acting on the charges.

The electroscope.—The most convenient apparatus for

examining electric charges is the electroscope, of which a simple type is shown in Fig. 32. A stout brass or copper wire AC carries at its lower extremity a pair of leaves D, consisting of very thin metallic foil, either gold or aluminium. The conducting system ACD is insulated by means of the block B of paraffin wax or sulphur. If a charge of electricity be given to A it spreads over the system ACD, and the leaves D being now charged with electricity, the charges being of the same sign, there is a repulsion between them. The leaves being very light and flexible stand apart as shown in the diagram. If A be touched by hand, or "earthed," the leaves collapse because the charge escapes through the hand to earth. The leaves may be given a negative charge by contact with rubbed ebonite, or a positive charge with rubbed glass, although the most convenient way of charging them is that described on p. 44.

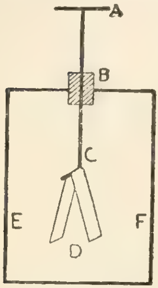


FIG. 32.—Electroscope.

Conductors and insulators.—The electroscope affords a convenient means of comparing the conducting powers of poor conductors. If the leaves be charged and the cap A be touched by a metallic wire held in the hand, the leaves collapse instantly on account of the high conducting power of the wire. But if a piece of dry cotton be used, the leaves will collapse slowly because the cotton is a poor conductor of electricity. If the cotton is moistened, the leaves collapse almost as rapidly as if a metal wire were used. Similarly, dry wood is a poor conductor.

If a piece of paraffin-wax, sulphur, or amber be used, the charge will hardly leak away at all. Such bodies are called non-conductors or **insulators.** The student should compare the insulating powers of various substances such as glass, sealing-wax, paper, oiled paper, etc. in this way.

Detection of kind of charge.—If the electroscope be given a positive charge, and a positively charged body be brought near it, the leaves will **increase in divergence,** while if a negatively charged body be brought near it, the leaves will **decrease in divergence.** The reason for this will be given later (p. 45), but the fact may be used to discover the sign of

the charges on various bodies when rubbed, and the signs of the charges produced on the rubbers. A warning, however, is necessary, for if the hand, or any uncharged conductor, be brought near the electroscope, there will be a slight collapse of the leaves on this account alone.

Potential.—It has already been seen (p. 38) that the presence of an electric field implies that positive electric charge experiences a force driving it in a particular direction, while negative charge experiences a force in the opposite direction. If the field causes a movement of the charge, work is performed, for according to the laws of dynamics,

$$\textbf{Work} = \textbf{Force} \times \textbf{distance.}$$

If the force is measured in dynes and the distance in centimetres, the work done is measured in **ergs**; for the C.G.S. unit of work, or the work done when a force of one dyne produces motion through a distance of one centimetre, is the C.G.S. unit of work and is called the **erg.**

In the case of electricity, the force is due to the presence of an electric charge in an electric field, but it is still measured in dynes, so that the work done when a charge is moved may still be measured in ergs. It is, therefore, possible to represent an electric field, not by the force upon unit positive charge, but by the work done upon unit positive charge when moved from one place to another. **The work performed when a unit positive charge is moved from one point to another is called the difference of potential between the two points.**

Calculation of potential due to a single charge.—If a charge of q units of electricity be situated at O in Fig. 33, and a unit

FIG. 33.—Calculation of potential.

positive charge be placed at A, it will experience a force driving it towards B. The force on the unit charge when situated at A is q/a^2, if a is the distance OA. Similarly the force on it when at B is q/b^2, and the force at each of a number of intermediate points such as PRS . . . T, may be found. The total work done on the charge as it moves from A to B must be found by taking

the work for each of the small steps and adding to get the total. If we make a diagram as in Fig. 34, and set up AC to scale equal to q/a^2, PD equal to q/p^2, etc., we obtain a curve CDE and each of the small strips such as APDC has an area, to scale, equal to the work done in moving the unit charge from A to P.

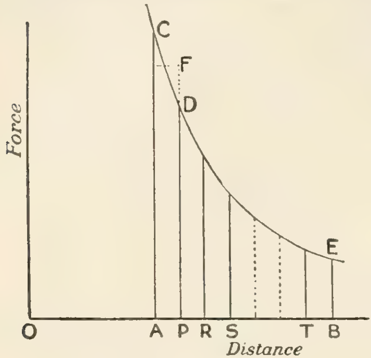


FIG. 34.—Force diagram.

For, work=average force $\times$ distance.

The average force is represented by the average of AC and PD and the distance=AP=$(p-a)$. The average height PF is—

$$\frac{1}{2}\left(\frac{q}{a^2}+\frac{q}{p^2}\right)=\frac{(a^2+p^2)q}{2a^2p^2}$$

If, now, the strips are made extremely narrow so that AP is very small, and a very nearly equal to p, we get nearer and nearer to the condition $a-p=0$, or,

$$a^2+p^2-2ap=0$$

or,

$$\frac{a^2+p^2}{2}=ap$$

so that if the steps are extremely small, the average force over the range AP is q/ap, and—

$$\text{Work done in moving unit charge from A to P}=\frac{q}{ap}(p-a)$$

$$=\frac{q}{a}-\frac{q}{p}$$

Similarly, ,, ,, ,, ,, P to R$=\dfrac{q}{p}-\dfrac{q}{r}$

,, ,, ,, ,, R to S$=\dfrac{q}{r}-\dfrac{q}{s}$

. . . .

,, ,, ,, ,, T to B$=\dfrac{q}{t}-\dfrac{q}{b}$

On adding all these amounts of work together we see that the total is $\frac{q}{a}-\frac{q}{b}$. This is therefore the amount of work done on the unit positive charge on account of the presence of the charge $+q$, when it is moved from A to B.

A is said to be at a higher potential than B, because a positive charge is driven from points of higher to points of lower potential, that is, down the grade of potential. A negative charge would, of course, travel in the opposite direction, if free to move, that is, up the grade of potential.

Zero of potential.—Strictly speaking there is no zero of potential; only differences of potential between one point and another being found by the calculation. It is very convenient, however, to assume some zero of potential to which other potentials may be referred, just as heights are usually measured from the sea-level. There is no such thing as absolute height upon the earth, all we can measure is the difference of level between two points, and the sea-level is the most convenient zero to take. We may then call heights above the sea-level positive and those below negative. In a similar manner it is convenient to choose some zero of electrical potential and to call potentials above it positive, those below negative.

For some purposes, the earth is considered to be at zero potential, and a body connected to earth by a conductor will then acquire zero potential.

For other purposes it is convenient to consider all points at infinity to be at zero potential, because they are at an infinite distance from all the electric charges with which we have to deal. In this case, we should consider the potential of the point A (Fig. 33) to be q/a, because if B is imagined to be removed to an infinite distance its potential becomes zero, and b becomes infinitely great so that $\frac{q}{b}=0$.

Charging by influence.—By considering the potential of various points in an electric field, many phenomena may be explained. It must be remembered that the presence of a positive charge of electricity raises the potential of all points in its neighbourhood, the nearer points being raised more in potential than more distant points. Also a negative charge in a similar manner lowers the potential of all points in its neighbourhood. And further, a positive charge experiences a force driving it from points of higher to points of lower potential. If, then, the two points are upon a conductor the charge is free to move along the conductor and will do so as long as the difference of potential exists.

Consider a metal conductor AB carried upon an insulating stand (Fig. 35 (*a*)) and bring a rubbed glass rod C near the end A. The presence of the positive charge on the glass raises the potential of the whole of AB, but the part A, being nearer to the positive charge, is raised more in potential than the distant part B. There will then be a current of positive electricity from A to B, down the grade of potential, and a current of negative electricity up the

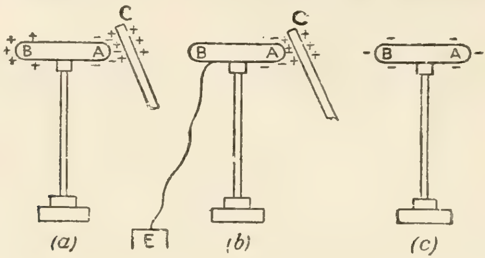


FIG. 35.—Charging by influence.

grade of potential. This flow will continue until the accumulation of positive charge at the end B raises its potential, and the accumulation of negative charge upon the end A lowers its potential to such an extent that all parts of the conductor are at one and the same potential. The flow will then cease, for, **when there is no movement of electricity upon a conductor, all parts of it must be at the same potential,** otherwise the electricity would move.

Now connect the conductor to earth, by a wire or by touching it by hand. Since the conductor is at a higher potential than earth, on account of the presence of the positive charge C (Fig. 35 (*b*)) positive charge will flow to earth until the potential of the conductor is reduced to that of earth. The conductor therefore loses positive charge and now possesses a balance of negative charge.

Next break the earth connection (Fig. 35 (*c*)). Now charge cannot escape. Then remove the positively charged rod C. The conductor falls in potential owing to the removal of the positive charge on C. It has now a negative charge and a negative potential. We say that the conductor has been **charged by influence.**

It should be noted that every body contains inexhaustible supplies of positive and negative electricity (see Chapter XIII), but in the usual neutral or uncharged state, the amounts of positive and negative electricity upon it are equal. When there is an excess of either kind the body is said to be charged.

Proof plane.—The state of electrification of the surface of a conductor can be investigated by means of a **proof plane.** This consists of a small thin disc of metal (Fig. 36), carried by an insulating rod of ebonite or varnished glass. If a conductor AB be given a positive charge by influence from a negatively charged ebonite rod, the density of the charge at various parts of its surface may be found by placing the proof plane in contact with it. While touching, the proof plane becomes part of the surface of the conductor,

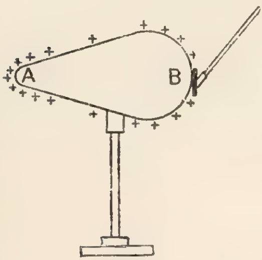


FIG. 36.—Distribution of charge on conductor.

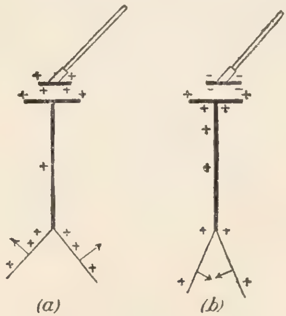


FIG. 37.—Determining sign of charge.

and on removing the proof plane it takes away with it part of the charge, the amount depending on the density of the charge at the place. The amount and sign of the charge on the proof plane may be found by bringing it near a charged electroscope. If the electroscope possesses a positive charge it has a positive potential, and the presence of a positive charge on the proof plane (Fig. 37 (*a*)) will raise the potential of the electroscope still more and the leaves will increase in divergence.

If, on the other hand, the charge on the proof plane is negative (Fig. 37 (*b*)), the potential of the electroscope is lowered and is brought nearer to zero potential and the divergence of the leaves will decrease. Had the electroscope been charged negatively the opposite effects would have been observed.

By this means, the student should examine the distribution of charge upon the conductor shaped as in Fig. 36, when it will be

found that the more pointed parts such as A will have a far greater density of charge upon them than the less curved parts. Also the nature of the charges during the process of charging by influence (Fig. 35) should be investigated by means of the proof plane and electroscope in a similar manner.

Discharge from points.—From the conductor shown in Fig. 36 it is seen that the density of charge is greatest at the more pointed parts. If a conductor carries a very sharp point such as a needle, the accumulation of the charge at the point becomes enormous, and the strength of the electric field in the air near the point is so great that the air becomes a conductor of electricity (Chapter XIII). A current of air is then driven away from the point and carries with it electricity of the same kind as that on the point. By making a little spider as in Fig. 38, with the needle-points ABCD turned all one way, and balancing it on a pivot, it will be found on connecting it to an electrical machine (p. 51) that the current of air streaming from the points has momentum, and the reaction drives the spider round as shown.

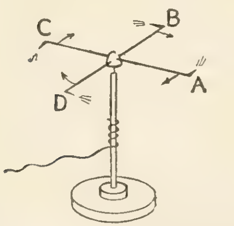


FIG. 38.—Discharge from points.

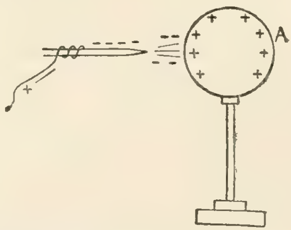


FIG. 39.—Discharging a conductor by means of a point.

A charged insulated conductor such as A (Fig. 39) may be discharged by directing towards it a needle-point connected to earth. The point becomes negatively charged by influence as described on p. 44, and the stream of negatively charged air proceeding from it will, on meeting the body A, neutralise its positive charge, that is, it will discharge it. It is upon this effect that the efficacy of lightning conductors depends. The lightning conductor consists of an earth-connected conductor ending in a number of points, attached to the highest part of the building to be protected. Any passing cloud which happens to be charged with electricity causes a streaming of electricity from the points of the lightning conductor. The opposite charges on cloud and building become

quietly neutralised in this way, whereas had there been no lightning conductor, they might have accumulated until the air insulation broke down suddenly, with production of an enormous destructive current. For a similar reason pine trees, whose leaves are needle-like in form, are far less often struck by lightning than oak trees, whose leaves are of a blunt, rounded form.

Hollow conductors.—The electrical condition of the inside of a conductor is of importance and may be investigated by the use of hollow conductors.

Experiment (i).—The conductor may be a tin can or any vessel made of metal sheet, and it must rest upon an insulating stand, which may be a block of paraffin wax, B, as in Fig. 40. On charging the can, either positively or negatively, it will be found that charge may be taken from the outside surface by means of a proof plane. On attempting to take charge from the interior surface by means of the proof plane, the attempt will fail because **there is no charge on the inner surface of a hollow conductor.**

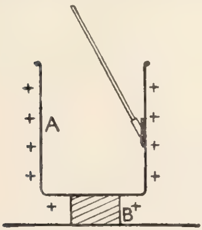


FIG. 40.—Zero charge inside hollow conductor.

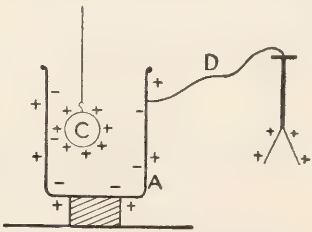


FIG. 41.—Faraday's ice-pail experiment.

Experiment (ii).—Next, let a body C, carried by a silk thread, be charged and lowered into the can A (Fig. 41), the can being connected to the electroscope by a wire D. The can and electroscope together form a conductor corresponding to AB in Fig. 35. The potential is raised by the approach of the positive charge on C and the leaves diverge. If C is allowed to touch the interior of C there will be no motion of the leaves; if C be withdrawn it is found to be completely discharged. The positive charge on C has exactly neutralised the negative charge produced on the inside of the can, and the positive charge remaining on the can and electroscope must therefore be exactly equal to the charge originally on C, because the can and electroscope were originally uncharged. This method affords a convenient means of making

a charged body such as C give up the whole of its charge to a conducting body attached to the electroscope.

Experiment (iii).—Let the can A be charged, say positively, and the conductor C charged by influence from the charge upon A. There is no difficulty when C is outside the can A, but when it is inside as in Fig. 42, let it be touched momentarily by an earthing wire. On withdrawing C and placing it inside the can B it will

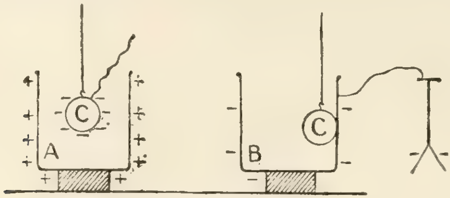


FIG. 42.—Faraday's ice pail experiment.

give up its whole charge to the can B and electroscope. Thus the body may be charged by influence when situated inside a hollow charged conductor, which shows that the space inside the hollow conductor is at a potential which depends upon the charge upon the conductor.

If, in addition, the experiment be repeated with C at various positions inside the hollow charged conductor A, at the moment of being earthed, it will be found that the same amount of charge, as indicated by the divergence of the leaves of the electroscope, will always be acquired, provided that C is not too near the entrance of the can A. It follows that **the whole of the space inside the hollow conductor is at the same potential,** for the charge acquired on earthing any given body depends upon the potential of the space in which it is situated at the moment of being earthed.

Since the presence of an electric field is necessarily accompanied by a variation of potential from point to point in the field, and the points inside a hollow conductor are all at the same potential, it follows that **there is no electric field inside a hollow conductor.**

These experiments were first conducted by Michael Faraday, and very important conclusions are drawn from them. Owing to the fact that he used ice pails for the hollow conductors, they are usually known as **Faraday's ice-pail experiments.**

Electrophorus.—We will now consider several appliances for producing electric charges in quantity, usually known as electrical machines. They depend for the multiplication of charge upon the process of charging by influence (p. 44) and are called **influence machines,** although in all cases the first charge is produced by friction. The simplest of these is the **electrophorus** (Fig. 43), which is merely a sheet of ebonite A which has been rubbed with fur and therefore negatively charged, and a brass plate B which rests upon it. B may be lifted by its insulating handle, but before lifting it is earthed by touching and is therefore positively charged. This positive charge may be used as required, and the process repeated by replacing and earthing. Sometimes a brass sole plate S is used, a metal pin passing through the ebonite and touching, and so earthing, the plate B when placed on the ebonite, thus saving the necessity of earthing it by hand. The negative charge remains on the ebonite for a long time as it is only removed to a very small extent each time the plate B touches the ebonite, since the ebonite is a good insulator and is only touched by the brass plate at a few points on contact being made.

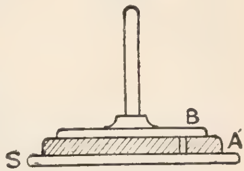


FIG. 43.—Electrophorus.

Thus an almost inexhaustible supply of positive charge is obtained and it might be thought at first sight that the energy associated with this charge was obtained from nowhere. This cannot be true, and we can explain the presence of this energy by remembering that as the brass plate with its positive charge is lifted from the ebonite plate with its negative charge, there is an attraction between the two charges and work must be done in separating them. This work is the origin of the energy possessed by the positive charge. In other words, the plate when earthed comes to zero potential, and in carrying it away from the negative charge on the ebonite it is, of course, raised in potential; it is therefore carried up the grade of potential and work is done.

Kelvin replenisher.—The modern form of influence machine is developed from the Kelvin replenisher, a little machine devised by Lord Kelvin for the production of small amounts of electric charge. Two curved conductors A

and B (Fig. 44) must one or both be charged to a small amount to begin with. The conducting carriers C and D are fixed to an insulating arm E which can rotate about an axle, and carry C and D with it. While C is still near A and D near B, they are connected together electrically by the light springs F and G joined by a wire. Owing to the charges on A and B, C is then at a higher potential than D and a current flows from C to D, leaving C negatively charged and D positively. As the rotation continues, C comes in contact with the light spring S_2 connected to B, and D comes in contact with S_1. Since A and B are curved they act somewhat as hollow conductors and the charges of D and C are given up to them and therefore increase the charges they already possess. The process is then continued as the rotation goes on, the charges, positive upon A and negative upon B, being built up until the required potential is reached. The student may show by a similar reasoning that if the direction of rotation is reversed, the charges on A and B will be decreased.

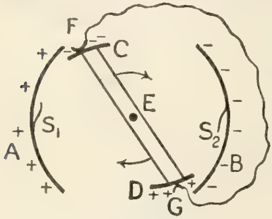


FIG. 44.—Kelvin replenisher.

Wimshurst machine.—Many forms of influence machine have been devised since the Kelvin replenisher, but probably the commonest and most efficient is the Wimshurst machine. This machine resembles the replenisher in several respects. If the fixed plates A and B (Fig. 44) be imagined to rotate in the opposite direction to the carriers C and D, there would

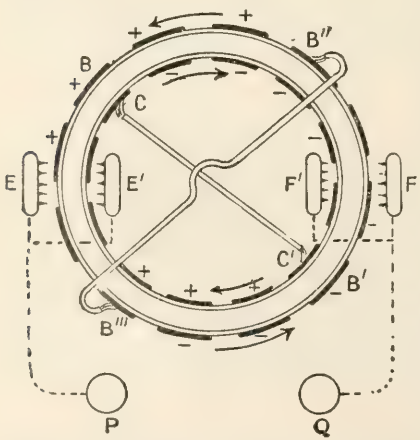


FIG. 45.—Diagram of Wimshurst machine.

then be four chargings per revolution instead of two. This is realised in the Wimshurst machine, and in addition the number of carriers and plates is increased. In Fig. 45 the Wimshurst machine is shown diagrammatically and B, B′, B″, B‴ correspond to the plates, and C, C′ to the carriers. These are actually strips of tinfoil or brass attached to two flat plates which in Fig. 45 are drawn as circles in

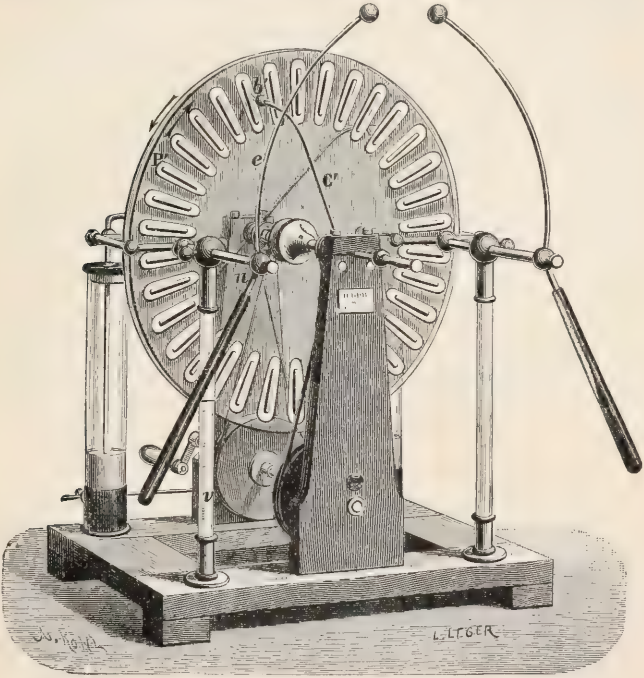


FIG. 46.—Wimshurst machine.

order to show the electrical operations involved more clearly. A view of the actual machine is shown in Fig. 46.

Returning to Fig. 45 we may trace the operations by following the sectors carried by the plates with the directions of rotation shown by the arrows. Imagine a positive charge to be situated on the sector B. At the instant that C passes B, C′ passes B′,

and C and C′ are connected by a wire carrying a brush at each end to make contact with the sector. Owing to the charge on B, C is at a higher potential than C′ and a current flows from C to C′, and C′ becomes positively charged while C is negatively charged. C then passes on and comes opposite to B″ as C′ comes opposite to B‴, and by a similar process B″ becomes positively charged and B‴ negatively. There are thus two streams of positive charges continuously carried towards the collectors E, E′, and two streams of negative charges towards the collectors F, F′.

The method by which the charges are collected from the sectors may be understood from the experiment described on p. 46. E and E′ are provided with points facing the sectors, and are also connected to the discharging knob P. Negative electricity streams from the points on to the positively charged sectors, neutralising the positive charge on the sectors and leaving the collector and the knob positively charged. In a similar manner the collector F and the knob Q become negatively charged. When these charges accumulate until there is a sufficiently high difference of potential between P and Q, a long spark discharge takes place between them, the current between them causing the disappearance of the charges.

In order to increase the current at discharge, condensers are attached to P and Q which allow a greater accumulation of charge before the discharging potential is reached. The function of the condenser will be understood better after reading the next chapter.

Exercises on Chapter IV

1. Describe how you would show that electrification of different kinds may be produced by rubbing together bodies of various natures.

2. Explain how you would show that like kinds of electricity repel each other, and that unlike kinds attract each other.

3. State the law of force between charges of electricity, and deduce a definition of unit charge of electricity upon the C.G.S. system.

4. A charge of 15 units of positive electricity is situated midway between two charges, 12 cm. apart. Find the force on the 15 units when the two charges are (*a*) $+30$ and -20 units respectively, and (*b*) $+30$ and $+20$ units.

5. Two charges of electricity respectively $+200$ and -200 C.G.S. units are 15 cm. apart. Find the strength of electrical field at a point situated at a distance of 10 cm. from each.

6. Sketch approximately the electric lines of force for a positively charged conducting sphere situated at the middle point of a cubical room.

7. Describe the electroscope and show how you would identify the two kinds of electricity by means of it.

8. Give an account of an experiment for comparing the insulating powers of different materials.

9. Define the difference of electrical potential between two points. What is the difference of potential between two points A and B, if A is situated 30 cm. and B 12 cm. from a positive charge of 180 C.G.S. units?

10. Show how to calculate the difference of potential between two points at different distances from a given charge of electricity.

11. Describe in detail the process of charging a conductor by influence.

12. Describe how you would investigate the distribution of charge upon a conductor.

13. How would you show that any charge residing upon a hollow conductor is entirely upon the outside.

14. Describe an experiment which proves that the whole of the space within a hollow charged conductor is at the same potential.

15. Give an account of the electrophorus, and explain from where the energy of the charge obtained is derived.

16. Two bodies are separately charged. How would you find out by experiment which of the bodies possesses the greater charge.

17. Describe some form of influence machine for the continuous production of electricity.

18. Charges of $+9$, $+15$, and -6 units respectively are placed one at each corner of an equilateral triangle whose length of side is 6 cm. Find the potential at the mid-point of the triangle.

19. In the last question, find the work done in carrying a charge of 12 units from the mid-point of the triangle to a point midway between the $+9$ and the -6 charges.

20. A charge of 25 units is situated at a point whose distance from a charge of 40 units is 10 cm. Calculate the work done when the 25-units charge is moved to a distance of 40 cm. Also show that if the 40-units charge had been moved to a distance of 40 cm. the same work as before would have been done.

CHAPTER V

CAPACITY, ETC.

Capacity.—In the previous chapter several instances have been mentioned in which the potential of a conductor has been changed by placing a charge upon it. Now there must certainly be some relation between the potential produced by the charge and the quantity of charge itself. Remembering that the potential of a conductor is the work done in bringing a unit positive charge from infinity up to the conductor, and that the force upon the unit charge at each step of its path is proportional to the charge on the conductor, it follows that the potential of the conductor is proportional to the charge upon it. Of course it may have potential due to neighbouring charges, but that does not concern the present argument. Thus, the ratio of the charge on a conductor to the potential of the conductor due to that charge is a constant quantity, which depends upon the size and shape of the conductor itself. This ratio of charge to potential produced by the charge is called the **capacity** of the conductor.

If c is the capacity, v the potential, and q the charge, then, $c=\frac{q}{v}$, or, $q=cv$.

Capacity of a conducting sphere.—In certain simple cases the capacity of a conductor can be calculated. Consider a conducting sphere of radius a cm. at a distance from all other conductors and charges (Fig. 47), and let a charge $+q$ C.G.S. units of electricity be placed on it. Under these conditions the charge becomes uniformly distributed over the sphere, and the force on any charge outside the sphere due to q is exactly the same as though the charge q were concentrated at the

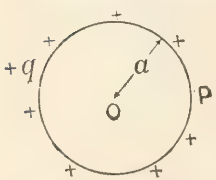


FIG. 47.—Charged sphere.

centre O of the sphere. The potential v at a point P very close to the sphere is therefore q/a (p. 43), or, $v = \frac{q}{a}$.

From the definition of capacity on p. 54 it follows that—

$$c = \frac{q}{v} = \frac{q}{\frac{q}{a}} = a$$

Thus the capacity of a sphere in C.G.S. units is equal to its radius in centimetres, or a unit of positive electricity placed on a sphere of one centimetre radius will raise its potential by one electrostatic C.G.S. unit Of course, had the charge been negative the potential would have been lowered by a similar amount.

Concentric spheres.—If the sphere of radius a cm. is surrounded by a concentric sphere of radius b cm. connected to earth, the case presented is of special importance (Fig. 48). The outer sphere is charged negatively by influence (p. 44), and the presence of this negative charge lowers the potential of the inner sphere. From Experiment (ii) (p. 47) we know that the negative charge on the earthed sphere is equal to $-q$. This produces a negative potential at its own surface, equal to $-\frac{q}{b}$.

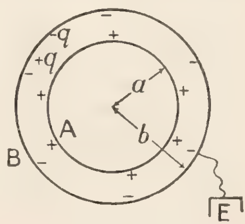


FIG. 48.—Spherical condenser.

But Experiment (iii) (p. 48) showed that the potential throughout a hollow charged conductor due to the charge upon it is uniform, and it follows that the potential throughout the space inside B and due to its negative charge is $-\frac{q}{b}$. The potential of A is therefore the algebraic sum of the potential $+\frac{q}{a}$ due to its own charge, and $-\frac{q}{b}$ due to the charge on B.

$$\therefore \text{ Potential of A} = \frac{q}{a} - \frac{q}{b}$$

$$\text{Capacity of A} = \frac{q}{\frac{q}{a} - \frac{q}{b}}$$

$$= \frac{ab}{b-a} \text{ C.G.S. units}$$

The effect of the earthed sphere is to increase the capacity of the insulated sphere, and it will be seen that the presence of an earthed conductor is always **to increase the capacity** of an insulated conductor. For, the charge produced by influence is of opposite sign to the charge on the insulated conductor, and therefore it brings its potential nearer to zero, so that the original charge does not change the potential so much as when the earthed conductor is absent. That is, the capacity of the insulated conductor is increased.

Parallel plate condenser.—The arrangement of the earthed conductor near an insulated conductor is called a **condenser,** because the charge acquired is greater for a given potential than without the earthed conductor. The above case of the spherical condenser is not of great practical value because of the difficulty of construction of truly concentric spheres, and the inner sphere is not accessible from outside. We can, however, deduce the capacity of a much more practical form, from the case of the spherical condenser whose capacity is $\frac{ab}{b-a}$. If we write $b-a=t$, the thickness of the space between the spheres,

$$\text{capacity} = \frac{ab}{t}$$

If, now, the spheres be imagined to increase in radius, keeping constant value of t, when the radius is very great compared with t, ab becomes sensibly equal to a^2,

and, $$\text{capacity} = \frac{a^2}{t}$$

$$= \frac{4\pi a^2}{4\pi t} = \frac{\text{area}}{4\pi t}$$

since $4\pi a^2$ is the area of the insulated sphere.

If, now, the radius goes on increasing, the surface eventually becomes plane, and since the capacity is uniformly distributed over the conductors, we see that for any pair of

parallel plates, one of which is insulated and the other earthed, the capacity is given by—

$$c = \frac{A}{4\pi t}$$

where A is the area of the insulated plate.

The effect of the earthed plate upon the capacity of the insulated plate may be shown by a simple experiment. Let the insulated plate A (Fig. 49) be charged and connected to the electroscope. On bringing the earthed plate B nearer to A the leaves of the electroscope collapse, because the capacity of A is increased, and the charge upon it no longer raises it to so high a potential as before. Thus, decrease of t increases the capacity. On the other hand, if t is increased by moving the earthed plate farther away from A, the leaves diverge more because the potential of A rises owing to its decreased capacity, the charge upon it being constant.

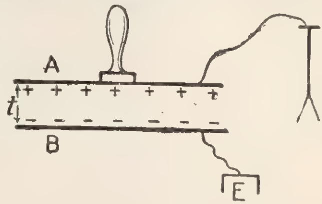


FIG. 49.—Parallel plate condenser.

Guard-ring condenser.—In the spherical condenser, the electric field and the lines of force are radial, that is, they are perpendicular to both spheres. As the spheres are imagined to grow to infinite radius this condition still holds, but when part of the infinite sphere is taken as a parallel plate condenser, the field will no longer be uniform at the edges of the plates. This want of uniformity is seen at the edges of the plates in Fig. 50. If the condenser has considerable area, very little error is introduced by neglecting this edge effect, but it may, in any case, be eliminated by the use of a **guard-ring** G. This guard ring is not reckoned in the area of the plate A, but its effect is to remove the irregular part of the field to a position beyond the edge of the plate A. The electric field is therefore perpendicular to A all over its area, and the value, area/$4\pi t$, is then the correct one for the capacity.

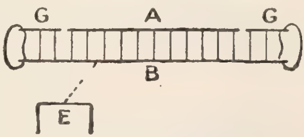


FIG. 50.—Guard ring condenser.

Variable condenser.—That the capacity of a condenser may be varied by changing the distance apart of the plates has been shown above. A much more convenient method

is adopted in practice, when a variable condenser is required; the effective area of the plates is changed, instead of their distance apart. In Fig. 51 is shown a common form of

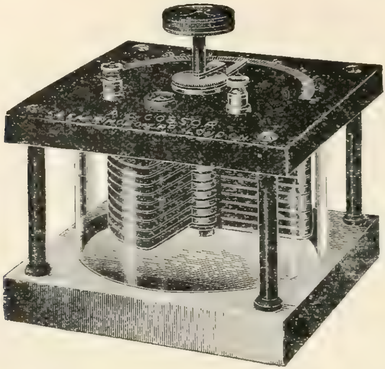


FIG. 51.—Variable condenser.

variable condenser of this type. The plates are of semicircular form, one set being fixed and the other set alternating between them and being attached to an axle so that by rotation, the area of the movable set opposite the fixed set can be varied. It will be seen that the capacity is great on account of the number of plates used. In the example shown there are ten movable plates placed respectively midway between eleven fixed plates. Such an arrangement is called a **multi-plate variable condenser.**

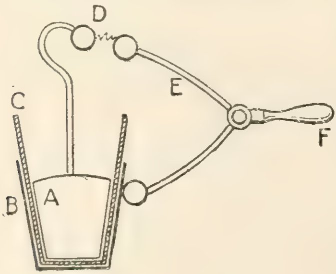


FIG. 52.—Leyden jar.

Leyden jar.—Another common form of condenser is the **Leyden jar,** which consists of a glass vessel or jar C (Fig. 52) with an outer coating of tinfoil, or sheet tin, and an inner coating A of a similar kind. A is the insulated coating and B the earthed coating. The coating B is usually effectually earthed by standing on the bench, or

by being held in the hand. The brass knob D is connected with the inner coating and enables contact to be made with it.

If the Leyden jar be charged by holding the knob D near the conductor P (Fig. 45) of the Wimshurst machine the inner coating will acquire a very great charge. On then standing the jar on the bench and connecting the inner and outer coatings by the brass discharging tongs E, held by an insulating glass handle F, a violent spark will be produced when the discharging current passes. Leyden jars are usually attached permanently to the conductors of the Wimshurst machine (p. 51) to increase their capacity, so that the discharging potential is not reached until a large quantity of electricity has accumulated. This makes a much heavier discharge than would be the case without the Leyden jars.

Dielectrics.—Up to now the electric charges have been considered to be situated in air. If, however, the medium separating the charges is not air, the force between the charges will not be the same. An instructive experiment for illustrating the importance of the medium may be performed with a Leyden jar whose coatings are removable. Let the jar be charged as usual, but instead of discharging it, remove the inside coating by lifting it out with the insulated discharge tongs and let the glass jar be lifted out of the outer coating. Make sure that both coatings are effectually discharged. Then put the Leyden jar together again, taking care to put in the inner coating by means of the insulating tongs. On connecting the coatings with the tongs, a spark, almost as violent as though the jar had not been taken to pieces, will be produced.

It is clear that the glass of the jar plays a more important part than that of merely holding the two coatings apart. When the coatings are charged, the electric field exists in the glass and it persists in the glass when the coatings are removed, provided the coatings are not discharged before removal. Any medium in which an electric field can persist is called **a dielectric.** Obviously the non-conductors are dielectrics, while the conductors are not dielectrics. For, an electric field in a conductor produces a current which flows until the redistribution of electric charge removes the field.

Dielectric constant or specific inductive capacity.—The

expression previously given (p. 37) for the force between electric charges is—

$$\text{Force} = \frac{q_1 q_2}{d^2} \text{ dynes}$$

This equation is only strictly true for a vacuum, but it is sufficiently near the truth for practical purposes when the charges are situated in air. If, however, the charges are situated in any other medium, the equation must be corrected by introducing a term which depends upon the medium concerned. Thus,

$$\text{Force} = \frac{q_1 q_2}{k d^2} \text{ dynes}$$

is true for all cases, if we take for k the quantity appropriate to the medium in which the charges are situated. The quantity k is called the **dielectric constant,** or sometimes the specific inductive capacity of the medium.

TABLE OF DIELECTRIC CONSTANTS.

Substance.	Dielectric constant (k).
Air	1·00058
Alcohol (ethyl)	26·8
Ebonite	2·8
Glass (crown)	5 to 7
,, (flint)	7 to 10
,, (plate)	5 to 7
Mica	5·7 to 7
Paraffin (oil)	4·7
,, (wax)	2 to 2·3
Olive oil	3·1
Quartz	4·5
Shellac	3·1
Sulphur	3·8
Water	80

From the above table it will be seen that the dielectric constant is greater than unity for all material media. For empty space its value is, of course, unity.

Potential in a dielectric.—On reviewing the reasoning on p. 42 by which the potential near a charge $+q$ was

found to be $+\frac{q}{a}$, it will be seen that on replacing the air by a dielectric whose constant is k, the force on the unit charge brought from infinity is q/kx^2 at distance x, instead of q/x^2. The distances are all unaltered by the change of medium, so that the work done for each step in the new medium is $1/k$ of the work when the medium was air. Therefore the potential, which is the work done on unit charge, is $1/k$ of its previous value. It follows that when the electric field exists in any medium, the potential v at distance d from charge $+q$ is given by

$$v = +\frac{q}{kd}$$

Effect of dielectric upon capacity.—Now the capacity of a conductor is the ratio q/v, where v is the potential to which the charge q raises the conductor. Hence, if the potential is only $1/k$ of its old value, when air is changed for a dielectric of constant k, the capacity will be k times the previous value.

The capacity of a sphere situated in a dielectric is therefore ka, where a is the radius.

The capacity of an insulated sphere of radius a surrounded by an earthed concentric sphere of radius b, and separated from it by a dielectric of constant k is $\frac{kab}{b-a}$.

The capacity of a parallel plate condenser with dielectric of constant k between the plates is

$$\frac{kA}{4\pi t}$$

It is now possible to calculate the capacity of a Leyden jar (Fig. 52) if the area of the inner coat and the thickness of the glass are known, the dielectric constant of the glass being taken as 6.

Useful condensers.—One of the most efficient forms of condenser for practical purposes consists of layers of tinfoil separated by sheets of mica (Fig. 53). Mica can be cleaved into very thin sheets and has a fairly high dielectric constant, both effects combining to give a high value to the capacity of the condenser. A further increase is effected by building up a

FIG. 53.—Large capacity condenser.

pile of sheets of tinfoil separated by mica, alternate sheets being connected to one terminal A and the others to B. In this way both sides of the sheets are effective in adding to the capacity.

The commoner type of condenser used when its value need not be known accurately is made by using paper impregnated with paraffin wax, instead of the mica, to separate the plates. In some cases a pair of sheets of tinfoil only are used, with a layer of paraffined paper between them and one outside, this compound sheet being folded or rolled up and pressed into a metal case.

A good form of standard condenser is shown in Fig. 54. The

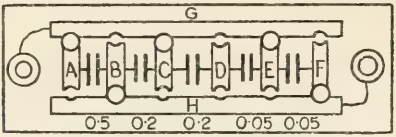


FIG. 54.—Standard condenser.

separate condensers are enclosed in a box and are only shown diagrammatically. They are connected to brass blocks A, B, C, etc., fixed on the ebonite lid of the box. By connecting the blocks with brass plugs to the long strips G and H, any desired combination of capacities can be used. With the plugs as shown, the capacity is 0·75 unit.

Condensers in parallel.—In order to understand how the effective value of the capacity can be obtained when several condensers are joined together, it must be noted that there are two common ways of making the connections between the condensers. When the condensers C_1, C_2, C_3 and C_4 (Fig 55) are connected with all their insulated plates to a common terminal P, and the earthed plates to a common terminal Q, they are said to be arranged **in parallel.**

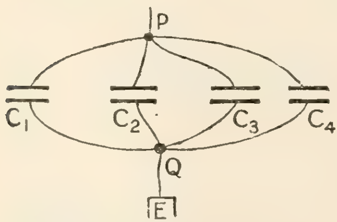


FIG. 55.—Condensers in parallel.

In order to find their effective capacity, note that the potential difference between the plates is the same for all of them; call it v. If q_1, q_2, q_3 and q_4 are the charges on the respective plates,

$$q_1 = c_1 v, \quad q_2 = c_2 v, \quad q_3 = c_3 v, \quad q_4 = c_4 v$$

And the total charge on all the plates being q, the combined capacity c is given by—

$$q = cv$$

But,

$$q = q_1 + q_2 + q_3 + q_4$$

$$\therefore \; cv = c_1 v + c_2 v + c_3 v + c_4 v$$

or,

$$c = c_1 + c_2 + c_3 + c_4$$

That is, **when condensers are arranged in parallel, the combined capacity is the sum of the separate capacities.**

Condensers in series.—When the condensers are connected together as shown in Fig. 56 they are said to be **in series,** and in this case the charge $-q$ on the second plate of the first, must be equal numerically to the charge $+q$ on the first plate of the second condenser, and so on. The effective capacity between A and D is then c, and is found by considering the potential difference between A and B to be v_1, between B and C, v_2, and between C and D, v_3. The whole difference of potential v between A and D is then—

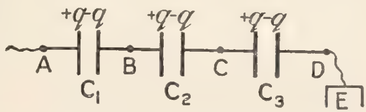


FIG. 56.—Condensers in series.

$$v = v_1 + v_2 + v_3$$

But,

$$v = \frac{q}{c}, \quad v_1 = \frac{q}{c_1}, \text{ etc.}$$

$$\therefore \; \frac{q}{c} = \frac{q_1}{c_1} + \frac{q_2}{c_2} + \frac{q_3}{c_3}$$

$$\therefore \; \frac{1}{c} = \frac{1}{c_1} + \frac{1}{c_2} + \frac{1}{c_3}$$

Thus the combined capacity is found by an addition of the reciprocals of the separate capacities.

As an example, let us find the effective capacity of condensers of capacities 5, 10 and 20, placed in series—

$$\frac{1}{c} = \frac{1}{5} + \frac{1}{10} + \frac{1}{20} = \frac{4+2+1}{20} = \frac{7}{20}$$

$$\therefore \; c = \frac{20}{7} = 2{\cdot}857$$

Energy of charge.—On bringing a charge to any place

from a distance, the work performed per unit of charge is equal to the potential of the place. Therefore, to bring the charge $+q$ units to a place where the potential is $+v$ units requires qv units of work. This work is recovered if the charge is allowed to pass down the grade of potential from the place considered to infinity.

If the charge $+q$ is brought from infinity and placed upon a conductor of capacity c originally at zero potential, the potential of the conductor rises to $+\frac{q}{c}$. In order to bring a further unit of charge from infinity, the work required is v or q/c, and on placing the unit on the body, the charge on it becomes $q+1$ and the potential $\frac{(q+1)}{c}$. On continuing to bring up unit charges, the potential continues to rise, and more work is done for each successive charge. Consider now the whole work done when a conductor is charged from zero potential and no charge, to a final potential v when the charge is q. Let the charge be brought up a very small amount at a time. If $1/n$ of a unit is brought at a time, then for the first step the potential is zero, and the work zero, but at the end of the step the potential is v/nq, since there are nq steps in the whole charging. The work for the second step is then v/n^2q, and the potential becomes $2v/nq$. For the third step the work is $2v/n^2q$ and the potential becomes $3v/nq$. This process continues, and for the last step the potential is $\frac{(nq-1)v}{nq}$, and the work done for this step is $(qn-1)/n^2q$. This completes the charge of q units by nq steps, $1/n$ of a unit at a time. The work for the first step is zero, and for the last step $\frac{nqv}{n^2q}=\frac{v}{n}$, if n is taken as such a large number, or $1/n$ such a small charge that $(nq-1)$ is sensibly equal to nq. This becomes nearer and nearer to the truth the smaller the charge taken at each step. Hence—

$$\text{Average work per step} = \frac{0+v}{2n} = \frac{v}{2n}$$

$$\text{Work for } nq \text{ steps} = \frac{nqv}{2n}$$

$$= \tfrac{1}{2}qv \text{ ergs.}$$

Remembering that $q=cv$, we may write the work required to charge the conductor as—

$$\frac{1}{2}qv = \frac{1}{2}cv^2 = \frac{1}{2}\frac{q^2}{c} \text{ ergs}$$

The work done in charging a conductor is performed in the reverse sense when the conductor is discharged. If the discharge takes place by driving the charges away a very small part at a time, the work is done upon the carrier of the small charges. As a rule, however, the discharging takes place by connecting the conductor to earth by a wire. The work done in driving the charge through the wire takes the form of heat and the wire is warmed. Sometimes a spark occurs when the circuit is nearly complete, and most of the heat then appears in the spark. That the temperature of the spark is very high can be recognised from the fact that a spectrum of the light emitted by the spark exhibits the lines characteristic of the volatilised metal of the contacts, as well as the spectral lines of the gas at high temperature, through which the spark is passing.

Sharing of charge between two conductors.—Consider two conductors, one of capacity c_1 having charge $+q_1$, and the other of capacity c_2 with charge $+q_2$. The potential of the first is $+q_1/c_1$ and of the second $+q_2/c_2$. If q_1/c_1 is greater than q_2/c_2, then on connecting the conductors by a wire, a current will flow from the first to the second, lowering the potential of the first and raising that of the second conductor until the two come to the same potential. Of course, if q_2/c_2 is greater than q_1/c_1, the current flows from the second conductor to the first.

In either case the total charge is q_1+q_2 and the total capacity c_1+c_2, so that the final potential is $\dfrac{(q_1+q_2)}{(c_1+c_2)}$. The total energy before the sharing of the charges is $\dfrac{1}{2}\dfrac{q_1^2}{c_1}+\dfrac{1}{2}\dfrac{q_2^2}{c_2}$, and the energy after sharing is $\dfrac{1}{2}\dfrac{(q_1+q_2)^2}{c_1+c_2}$. If the energy before sharing is greater than that after sharing, the loss of energy on sharing is

$$\frac{1}{2}\frac{q_1^2}{c_1}+\frac{1}{2}\frac{q_2^2}{c_2}-\frac{1}{2}\frac{(q_1+q_2)^2}{c_1+c_2}$$

$$=\frac{1}{2}\frac{q_1^2}{c_1}+\frac{1}{2}\frac{q_2^2}{c_2}-\frac{1}{2}\frac{q_1^2+q_2^2+2q_1q_2}{c_1+c_2}$$

$$=\frac{1}{2}q_1^2\left(\frac{1}{c_1}-\frac{1}{c_1+c_2}\right)+\frac{1}{2}q_2^2\left(\frac{1}{c_2}-\frac{1}{c_1+c_2}\right)-\frac{1}{2}\frac{2q_1q_2}{c_1+c_2}$$

$$=\frac{1}{2}q_1^2\frac{c_2}{c_1(c_1+c_2)}+\frac{1}{2}q_2^2\frac{c_1}{c_2(c_1+c_2)}-\frac{1}{2}\frac{2q_1q_2}{c_1+c_2}$$

$$=\frac{1}{2c_1c_2(c_1+c_2)}(c_2^2q_1^2+c_1^2q_2^2-2c_1c_2q_1q_2)$$

$$=\frac{(c_2q_1-c_1q_2)^2}{2c_1c_2(c_1+c_2)}$$

Whatever the values of the charges and capacities, this quantity must be positive because the numerator is a square and must always be positive, and capacities are necessarily positive. Hence there is always a loss of energy on sharing the charges, which loss takes the form of heat in the conducting wire, or the spark. In one case there may be zero loss; that is, when the numerator of the fraction is zero. In this case—

$$c_2 q_1 = q_2 c_1$$

or,

$$\frac{q_1}{c_1} = \frac{q_2}{c_2}$$

It is clear that when this condition holds, the conductors are both at the same potential before being connected together, and then there will, of course, be no current in the connecting wire.

Quadrant electrometer.—The gold-leaf electroscope is a useful instrument for measuring, roughly, differences of potential, and although it has in recent times been so improved that it may be looked upon as an instrument of precision, still, a more accurate measuring instrument is required. This was devised by Lord Kelvin and takes the form of four hollow quadrants A and B (Fig. 57), with a delicately suspended vane or paddle C, sometimes called a "needle," hanging inside them. This instrument is called a **quadrant electrometer.**

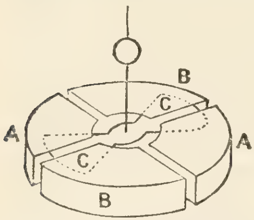


FIG. 57.—Quadrants and paddle of electrometer.

The paddle C is charged to a high potential, and if A and B are earthed, C takes some zero position determined by the suspending fibre. If, now, a difference of potential is established between A and B, the paddle C is rotated, the positive charge on C being urged down the grade of potential. The paddle comes to rest when the couple due to twist in the suspension is equal to the deflecting couple. Provided that the edges of the paddle lie well inside the hollow quadrants and that the outer edge of the paddle is circular in form, it may be shown by mathematical reasoning that—

$$\theta = K(v_a - v_b)\left(v_c - \frac{v_a + v_b}{2}\right)$$

where θ is the deflection, K a constant depending upon the

size of the instrument, and v_a, v_b and v_c the potentials of A, B and C. If the paddle is maintained at a potential which is very high with respect to v_a and v_b, the last factor in the equation is sensibly constant, and then the deflection is proportional to the difference of potential between the pair of quadrants A and the pair B.

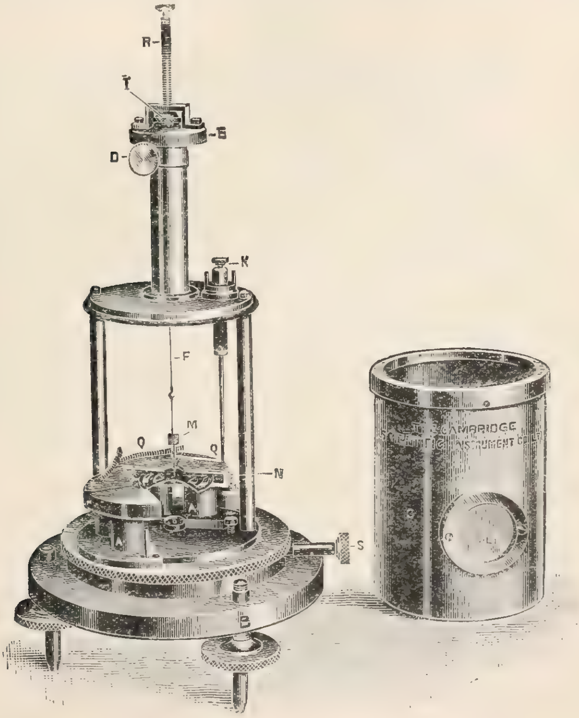


FIG. 58.—Dolezalek quadrant electrometer.

A modern form of the quadrant electrometer, much more convenient to use than the original Kelvin pattern, is that due to Dolezalek shown in Fig. 58, with the cover C removed. The quadrants Q are shown drawn apart to exhibit the paddle N. These quadrants are carried on amber pillars A, and opposite quadrants are connected together permanently

by light spirals of wire. Attached to the paddle support is a mirror M which enables the deflections to be measured, as described on p. 31. The paddle is suspended by a quartz fibre F hanging from the ebonite head E, by which the position of the zero is adjusted, which can be clamped by the screw D. The nut T and screw thread R enable the paddle to be raised or lowered to its proper position.

In the Kelvin quadrant electrometer, the potential v_c of the paddle is maintained constant by connecting it with the inner coating of a condenser which is charged by means of the replenisher (p. 50). But in the Dolezalek instrument the suspension F being a quartz fibre is such a good insulator that when once the paddle is charged from a battery by contact through K, it will remain at constant potential for a very long time.

If the paddle is connected permanently to the A quadrants, $v_c = v_a$, and the equation on p. 66 becomes $\theta = K^1(v_a - v_b)^2$; θ is then always in the same direction, since $(v_a - v_b)^2$ is always positive. The instrument connected up in this way will read alternating potential differences.

Use of the electrometer.—The electrometer is essentially an instrument for measuring differences of potential, but any measurement which can be made to depend upon differences of potential, such as the comparison of capacities, may be performed by it. Let us suppose that it is required to compare the difference of potential v_e between the poles of a cell E, with the difference of potential v_f between the poles of the cell F.

One pair of quadrants, say B (Fig. 59), is earthed by connecting to a water-supply pipe of the building, as is also the case of the electrometer, and the needle is charged by momentary contact with one pole of a battery of 50 cells, of which the other pole is earthed. A block of paraffin wax is procured and four holes, p, q, a and b, are bored in it and nearly filled with mercury. The quadrants A are connected to the hole a and the quadrants B to b. On placing a cross connector between a and b the paddle comes to rest and its zero position is noted. The terminals of the cell E are connected to p and q as shown, and on removing the a and b connector and putting wires to connect a and q, p and b, the terminals

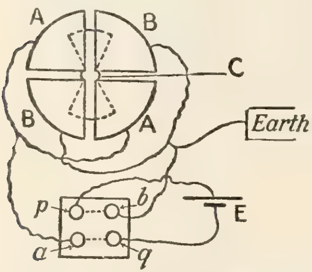


FIG. 59.—Comparing differences of potential.

of the cell are joined to the respective pairs of quadrants, and a deflection upon the scale will be noted.

On removing the connectors so that they join *a* and *p*, *b* and *q*, a reverse deflection will be observed, and the mean of the two (θ_e) is the deflection for the cell E.

The cell E is now replaced by the cell F, and the mean deflection (θ_f) is obtained in the same way.

Then,
$$\frac{v_e}{v_f}=\frac{\theta_e}{\theta_f}$$

Comparison of capacities.—In order to find the ratio of the capacity c_1 of one condenser to the capacity c_2 of another condenser, let the first be connected to the quadrant electrometer as shown in Fig. 60, and the condenser charged by momentary contact with the terminals of a cell. The deflection θ_1 is then observed, the potential difference between the plates of the condenser being v_1. The second condenser is now connected to the first as shown by dotted lines, and shares the charge of the first, and the new potential difference v_2 corresponds to the new deflection θ_2, or,

$$\frac{v_1}{v_2}=\frac{\theta_1}{\theta_2}$$

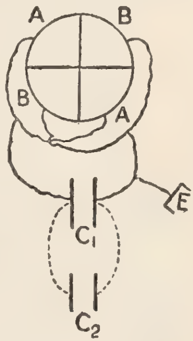


FIG. 60.—Comparing capacities.

On sharing the charge, a small charge q has passed from the first condenser to the second, and in so doing lowered the potential of the first by the amount v_1-v_2; and raised the potential of the second by the amount v_2.

$$\therefore \quad c_1=\frac{q}{v_1-v_2}, \text{ and, } \quad c_2=\frac{q}{v_2}$$

thus,
$$\frac{c_2}{c_1}=\frac{v_1-v_2}{v_2}$$
$$=\frac{\theta_1-\theta_2}{\theta_2}$$

From the two electrometer deflections, the ratio of the capacities can therefore be calculated.

Measurement of dielectric constant.—Since the capacity of a condenser depends upon the dielectric in which the electric

field exists, we have seen (p. 61) that the capacity is proportional to the dielectric constant of the medium between the plates of the condenser. If, therefore, the capacity of a condenser can be measured (c_a) with air as dielectric and again with some other medium as dielectric (c_m) it follows that the ratio (c_m/c_a) of the two capacities is the dielectric constant of the medium. For the present experiment we may take the dielectric constant of air as equal to unity. For this purpose two flat metallic plates may be used to form the condenser. The layer of silver at the back of a plate-glass mirror forms a very good plate for this purpose, the paint or varnish having been previously dissolved off by means of methylated spirit. Two such plates are required, and one of them is reduced to a circle of silver, A (Fig. 61), by scraping off the

FIG. 61.—Measurement of dielectric constant.

silver beyond the circle, while the silver, B, on the other plate is left entire. The plate A is placed face down over the sheet B, but separated from it by three pieces E F etc. cut from a sheet of ebonite or glass, or other material as required. The capacity (c) of the condenser AB is then found as described on p. 69 in terms of capacity of some convenient fixed condenser. The plate of dielectric from which E and F were cut is now placed between A and B, the pieces E and F being removed, and the capacity is again measured. If k is the dielectric constant of the medium, the new capacity is kc, and dividing the value last found by the first, k is obtained.

It should be noted that the silver is so thin that the distance between A and B in the two cases is approximately the same. If desired, the thickness of the silver may be determined by weighing a piece of the mirror with and without the silver, and a small correction made in the capacity measurement.

Faraday's condensers.—The first measurement of dielectric constant or specific inductive capacity was made by Faraday, using spherical condensers of the type shown in Fig. 62. Two such condensers were made, as nearly as possible alike, and their equality was tested by charging one and then connecting it to the second, the outer spheres being earthed. It was then found that the potential fell to half its previous value, as nearly as could be measured, and

it was concluded that the two were sufficiently near to equality. The lower half of one of them was then filled with shellac by running it in when molten, and then on comparing the capacities by sharing the charges as described on p. 69, it was found that the shellac had increased the capacity to 1·50 times its previous value. Remembering that only half the condenser was filled with shellac, we have—

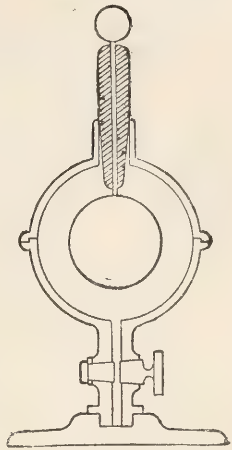


FIG. 62.—Faraday condenser.

$$k\frac{c}{2}+\frac{c}{2}=1{\cdot}50\,c$$

$$\therefore \quad k+1=3$$

or, $$k=2$$

This gave the value 2 for the dielectric constant of shellac.

Electric current as movement of charge.—In many of the experiments on electric charges, the movement of the charge has been spoken of as an electric current. This current is essentially the same in kind as that dealt with in Chapter III. It is convenient, however, to measure it in terms of the unit charge described on p. 37. Thus, a unit current flows in any conductor when unit charge passes any given cross-section in one second. A current measured in this way is said to be measured in **electrostatic units.** The unit of current given on p. 25 is derived from the unit magnetic pole, and a current measured in this way is said to be given in **electromagnetic units.** There is no confusion between the two systems of units as they are used in different classes of work, and the two units are not really independent of each other, because—

$$\left.\begin{array}{l}1 \text{ electromagnetic}\\ \quad\text{unit of current}\end{array}\right\}=3\times10^{10} \text{ electrostatic units of current.}$$

The quantity 3×10^{10} is the velocity of light in centimetres per second, and it was the inter-relation between the electrostatic and the electromagnetic systems of units that led James Clerk Maxwell to the electromagnetic theory of light.

Quantity of electricity on the electromagnetic system.—Since, in all cases, current is the electric charge passing per second, the electromagnetic unit of charge passes when the electromagnetic unit of current flows for one second. Thus,

$$\left.\begin{array}{l}\text{1 electromagnetic unit}\\ \text{of electric charge}\end{array}\right\}=\left\{\begin{array}{l}3\times 10^{10}\text{ electrostatic units of electric}\\ \text{charge.}\end{array}\right.$$

If instead of one electromagnetic unit of current flowing for one second, the current of one ampere (p. 75) be taken, then the amount of charge which passes is only one-tenth of the absolute unit. Such an amount is called **one coulomb,** and the coulomb may therefore be defined as the amount of electricity which passes in one second when a current of one ampere is flowing.

Thus 1 coulomb $=3\times 10^9$ electrostatic units of charge.

Difference of potential on the electromagnetic system.—From the unit difference of potential, defined on p. 41 as that existing between two points when the work done in carrying unit charge from one point to the other is one erg, we see that for the work to be still one erg if the enormous electromagnetic unit of charge is to be carried from one point to the other, the unit difference of potential on the electromagnetic system must be very small, in fact,

$$\left.\begin{array}{l}\text{1 electrostatic unit of}\\ \text{potential difference}\end{array}\right\}=\left\{\begin{array}{l}3\times 10^{10}\text{ electromagnetic units of}\\ \text{potential difference.}\end{array}\right.$$

Capacity on the electromagnetic system.—On the electromagnetic system, the unit of capacity is the ratio,—

$$\frac{\text{1 electromagnetic unit of charge}}{\text{1 electromagnetic unit of p.d.}}=\frac{3\times 10^{10}}{\frac{1}{3}\times 10^{-10}}=9\times 10^{20}$$

electrostatic units of capacity.

Using practical units, the coulomb and volt (p. 75), the unit capacity called the **farad.** Thus,—

$$\begin{aligned}1\text{ farad}=\frac{\text{1 coulomb}}{\text{1 volt}}=\frac{10^{-1}}{10^{8}}&=10^{-9}\text{ absolute e.m. units.}\\ &=9\times 10^{20}\times 10^{-9}\\ &=9\times 10^{11}\text{ electrostatic units.}\end{aligned}$$

The microfarad is 10^{-6} of this and is therefore 9×10^5 e.s. units. Hence the capacity of the condenser (p. 61) is

$$\frac{kA}{4\pi t}\cdot\frac{1}{9\times 10^5}\text{ microfarads.}$$

Exercises on Chapter V

1. Define "capacity" and show how to calculate the capacity of an insulated conducting sphere.

2. Two spheres, A of 10 cm. radius and B of 15 cm. radius, are situated some distance apart. A receives a charge of +30 C.G.S. units and B a charge of +40 C.G.S. units. Which way will a current flow if the two spheres are connected by a wire?

3. Show how to calculate the capacity of a conducting sphere surrounded by an earthed concentric sphere. If the radii of the spheres are 100 and 100·5 cm., what is the capacity of the inner sphere?

4. Calculate the capacity of a cylindrical Leyden jar if the inner diameter of the glass vessel is 16 cm., the height of the inner coating 18 cm., and the thickness of the glass 0·2 cm. (Take k for glass = 6.)

5. In the last question what will be the potential of the inner coating if a charge of 132 electrostatic units is placed on the inner coating of the jar, and what amount of energy is liberated when the jar is discharged?

6. Explain the effect on the capacity of an insulated conductor, of bringing an earthed conductor near it.

7. Show how to calculate the capacity of a parallel plate condenser. What is the capacity of a plate condenser of area 80 sq. cm. if a thickness 0·3 cm. of air separates the plates.

8. What is a dielectric? Explain the effect on the capacity of a conductor of replacing the air by another dielectric.

9. Prove that the energy of the charge upon a conductor is $\frac{1}{2}cv^2$.

10. The radii of two concentric spheres are 50 and 52 cm. respectively, the outer sphere being earthed, and a charge of 200 units is given to the inner sphere. Find the energy of the charge when the dielectric is air and also when it is a medium of dielectric constant 6·5.

11. Calculate the effective capacity of three condensers in series, whose capacities are respectively 8, 20 and 25 C.G.S. units. If the charge given to the first condenser is 15 units, what is the difference of potential between the first plate and earth?

12. A condenser of capacity 30 is charged to potential of 25 units. It is then connected to an uncharged condenser of capacity 20. What is the loss of energy when the charge becomes shared between the two condensers?

13. In the last question, what is the amount of charge which passes from one condenser to the other, and what is the common potential of the two condensers?

14. A parallel plate condenser A of area of plate 80 sq. cm., thickness of air space 0·5 cm., is charged with −30 units of electricity. A second condenser B has area 120 sq. cm., thickness of dielectric 0·8 cm., of constant 4·2, and is given a charge of −15 units. Which way will the current flow on connecting the two condensers by a wire, and what will be their common potential?

15. A condenser is connected to a quadrant electrometer, and the deflection is 74 scale divisions. The deflection falls to 35 scale divisions when a parallel plate condenser having air as dielectric is connected to the first. If a slab of glass had occupied the space between the plates of the second condenser the deflection would have fallen to 10 scale divisions. What is the dielectric constant of the glass?

16. A condenser of capacity 7500 electrostatic units is charged to a difference of potential of 3000 electrostatic units. What is the charge on the condenser, measured in electromagnetic units?

CHAPTER VI

OHM'S LAW

Rate of working.—Whenever electricity moves from place to place work is done. The movement takes place on account of the electric field, a positive charge being driven from points of higher to points of lower potential in the field. The work done upon the positive charge appears in different forms of energy according to the conditions applying. If the movement takes place upon a conductor, the energy appears in the form of heat in the conductor. If the charge resides upon a very small mass, the energy becomes kinetic energy of the mass (Chap. XIII.). In any case the current is measured by the amount of electricity passing per second, and we have seen that the amount of work done when a unit of electricity moves from one point to another is the difference of potential between the points.

Consider the difference of potential between two points A and B (Fig. 63) to be e. This means that when one unit of electricity moves from A to B the work done is e ergs. The current i corresponds to i units of electricity passing per second, and the rate at which work is performed when the current flows continuously is ie ergs per second.

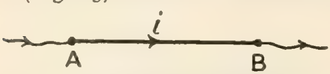


FIG. 63.—Current in conductor.

It follows that the unit difference of potential on the electromagnetic system exists between two points when unit current corresponds to a rate of working of one erg per second. It is usual to contract the term "difference of potential," or potential difference, into the initials p.d. for convenience Thus—

$$\text{Rate of working} = \text{current} \times \text{p.d. ergs per second}$$
$$= ie \text{ ergs per second}$$

In the following pages we shall employ the electromagnetic units, the electrostatic system only being used when specially stated.

Practical units.—Having derived the unit of current from the magnetic pole, and the magnetic pole from the dyne, which in turn is derived from the centimetre, gramme and second, it follows that the unit of current may not be of convenient size for the measurement of currents in practice. This is the case, and for practical purposes a unit is chosen which is only one-tenth of the absolute unit. This new practical unit is named after the celebrated experimenter André M. Ampère, and is called the **ampere.** If currents are measured in amperes instead of absolute units which are larger, it follows that most of the quantities calculated from them would be too large, and must be corrected by a factor 10. Thus, the magnetic field at the centre of a circular coil is $2\pi ni/r$ gauss where i is in absolute units, but it is $2\pi n\mathrm{I}/10r$ where I is measured in amperes.

In a similar way, the absolute unit of p.d. is much too small for convenience in practice, so a new unit is chosen which is 100000000 or 10^8 times as great. It is named after Alessandro Volta, the discoverer of the battery, being called the **volt.**

If the current in a conductor be I amperes when the p.d. between the ends of the conductor is E volts, then—

$$\text{Rate of working} = \frac{\mathrm{I}}{10} \times \mathrm{E} \times 10^8 \text{ ergs per second}$$

$$= \mathrm{I} \times \mathrm{E} \times 10^7 \text{ ergs per second}$$

A practical unit of rate of working called the **watt,** after James Watt the engineer, is taken as the rate of working corresponding to 1 ampere and 1 volt. It follows that—

$$\text{Rate of working} = \mathrm{I} \times \mathrm{E} \text{ watts}$$

and, $1 \text{ watt} = 10^7$ ergs per second

also $1 \text{ joule} = 10^7$ ergs; $\therefore$ $1 \text{ watt} = 1$ joule per second

It is of interest to note that the British unit of rate of working employed by engineers is the horse-power of 33,000 foot-pounds per minute. On converting this into ergs per second and into watts, it will be found that—

$$1 \text{ horse-power} = 746 \text{ watts}$$

As an example let us find the horse-power used in a lamp

carrying a current of 1·5 ampere, when the p.d. between its terminals is 220 volts.

$$\text{Rate of working} = 1{\cdot}5 \times 220 \text{ watts}$$
$$= \frac{1{\cdot}5 \times 220}{746} \text{ horse-power}$$
$$= 0{\cdot}442 \text{ horse-power}$$

Ohm's law.—For every conductor there must be some relation between the current flowing in it and the p.d. between its ends. Suppose that the conductor consists of a wire AB (Fig. 64) and that the current flowing through it be measured by means of a tangent galvanometer G. At the same time let the p.d. between the ends of the conductor be measured by means of the quadrant electrometer E. It will then be found that so long as the wire is kept under constant conditions, particularly as regards temperature, **the current is proportional to the potential difference.** This law was discovered by G. S. Ohm, and is called Ohm's law. It may be expressed as follows :—

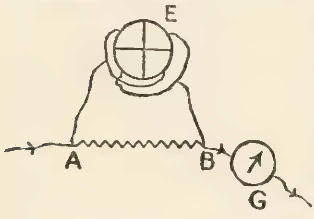


FIG. 64.—Proof of Ohm's law.

$$\frac{\text{P.D. between ends of conductor}}{\text{Current in conductor}} = \text{Constant}$$

Ohm's law applies to all metallic conductors, but it must not be taken as true for all cases in which there is a current. It applies also to the currents in liquids, but not to that in gases.

Resistance.—The particular constant in the above relation is only the same for any one conductor. It varies as we change the conductor and depends for its value upon the nature of the conductor. It receives a particular name, being called the **resistance** of the conductor. When the p.d. is unity for unit current flowing, the conductor has unit resistance. The practical unit of resistance is called the **ohm.** If E be measured in volts and I in amperes, then—

$$\frac{E}{I} = R \text{ ohms}$$

Similarly, $E = I \times R$, and, $\dfrac{E}{R} = I$

Again, $1 \text{ ohm} = \dfrac{1 \text{ volt}}{1 \text{ ampere}} = \dfrac{10^8 \text{ absolute units of p.d.}}{\frac{1}{10} \text{ absolute unit of current}}$

$= 10^9$ absolute units of resistance

Combination of resistances.—There are two principal ways in which conductors may be joined together in forming circuits. In the first, the same current flows through all the conductors. They are then said to be **in series** (Fig. 65).

FIG. 65.—Conductors in series.

Or, the current may divide between a number of conductors as in Fig. 66. They are then said to be **in parallel.**

(i) **Series.**—In order to find the combined resistance of a number of conductors in series, consider the arrangement of Fig. 65. Let R_1, R_2 and R_3 be the resistances of AB, BC and CD, and let the current I flow through them.

Then, p.d. between A and $B = IR_1$
,, ,, B and $C = IR_2$
,, ,, C and $D = IR_3$

$\therefore$ p.d. between A and $D = IR_1 + IR_2 + IR_3$

Also if R be the combined resistance between A and D,

p.d. between A and $D = IR$

$$\therefore \; IR = IR_1 + IR_2 + IR_3$$
$$R = R_1 + R_2 + R_3$$

Thus **when conductors are joined in series, the combined resistance is the sum of the separate resistances.**

(ii) **Parallel.**—If the conductors are joined in parallel as in

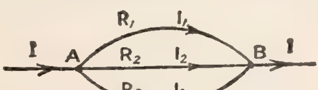


FIG. 66.—Conductors in parallel.

Fig. 66, the current entering at A divides into three parts, I_1, I_2 and I_3, which unite again at B to form the main current, and

$$I = I_1 + I_2 + I_3$$

If E is the p.d. between A and B, it is the same for all the conductors.

$$\therefore\ E = I_1R_1 = I_2R_2 = I_3R_3 = IR$$

where R is the effective resistance of the three conductors together.

$$\therefore\ I_1 = \frac{E}{R_1},\ I_2 = \frac{E}{R_2},\ I_3 = \frac{E}{R_3},\ \text{and } I = \frac{E}{R}$$

and,
$$\frac{E}{R} = \frac{E}{R_1} + \frac{E}{R_2} + \frac{E}{R_3}$$

or,
$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

The same argument applies, however many conductors there may be in parallel; **the reciprocals of the separate resistances must be added to obtain the reciprocal of the combined resistance.**

As an example:—What is the combined resistance when four conductors of resistances, 1 ohm, 5 ohms, 2 ohms and 10 ohms are arranged in parallel?

$$\frac{1}{R} = \frac{1}{1} + \frac{1}{5} + \frac{1}{2} + \frac{1}{10}$$

$$= \frac{10+2+5+1}{10} = \frac{18}{10}$$

$$\therefore\ R = \frac{10}{18} = 0{\cdot}555 \text{ ohm}$$

Current in one branch.—It often happens that it is necessary to find the current flowing in any one of the conductors in parallel. If the current in R_1 is required, note that—

$$I_1 = \frac{E}{R_1} = \frac{IR}{R_1}$$

Thus—

$$\text{Current in one branch} = \text{main current} \times \frac{\text{combined resistance}}{\text{resistance of branch}}$$

In the last problem, if the main current is 4 amperes, what is the current in the 2-ohm branch?

$$\text{Current in 2-ohm branch} = 4 \times \frac{0{\cdot}555}{2}$$

$$= 1{\cdot}11 \text{ ampere}$$

Specific resistance or resistivity.—The conductors used in practice are nearly always of regular form, such as a wire.

The resistance, then, must depend upon the length and area of cross-section of the wire, but it also depends upon the material of which the wire is made. In order to compare the behaviour of different materials, it is necessary to reduce the conductors all to the same dimensions. A conductor of unit length and unit area of cross-section is chosen for this purpose, the shape of the cross-section being immaterial. The resistance of this unit conductor is called the **specific resistance** or the **resistivity** of the material of which it is made.

TABLE OF RESISTIVITY.

Material.	Resistivity in ohms per unit conductor.	Temperature coefficient of resistivity.
Aluminium	$2{\cdot}94 \times 10^{-6}$ at 18° C.	0·0038
Copper	$1{\cdot}59 \times 10^{-6}$ at 18° C.	0·0043
Iron (soft)	$8{\cdot}85 \times 10^{-6}$ at 0° C.	0·0062
Lead	$2{\cdot}1 \times 10^{-5}$ at 18° C.	0·0043
Mercury	$9{\cdot}407 \times 10^{-5}$ at 0° C.	0·0009
Nickel	$1{\cdot}18 \times 10^{-5}$ at 18° C.	0·0062
Platinum	$1{\cdot}10 \times 10^{-6}$ at 18° C.	0·0038
Silver	$1{\cdot}54 \times 10^{-6}$ at 0° C.	0·0040
Manganin	$4{\cdot}43 \times 10^{-5}$ at 18° C.	0·00002
Platinoid	$3{\cdot}44 \times 10^{-5}$ at 18° C.	0·00025

Resistance of any conductor.—In order to find the resistance of a conductor which is not of unit dimensions, consider the resistivity of the material to be S ohms per unit conductor. Then to increase the length, the unit conductors

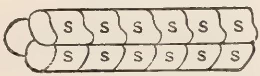


FIG. 67.—Resistance of conductor.

must be placed **in series**, and if there are l units of length the resistance is lS ohms. To increase the cross section the conductors must be placed **in parallel.** If the area of cross-section is to be A sq. cm. there must be A conductors in

parallel, all of resistance lS. To find the combined resistance we have—

$$\frac{1}{R} = \frac{1}{lS} + \frac{1}{lS} + \frac{1}{lS} + \ldots \text{ to A terms}$$

$$= \frac{A}{lS}$$

$$\therefore\ R = \frac{Sl}{A}$$

Or,

$$\text{Resistance of conductor} = \frac{\text{specific resistance} \times \text{length}}{\text{area of cross-section}}$$

In order to measure the resistivity of any material, a wire may be taken and its resistance measured (p. 98). On finding the length and cross-section, we have—

$$S = \frac{R \times A}{l}$$

from which the resistivity can be calculated.

Example.—Find what length of wire will have a resistance of 30 ohms, if the diameter of the wire is 0·22 mm. and the resistivity of the material 0·000048 ohm per unit conductor.

$$\text{Area of cross-section} = \ \times (0{\cdot}011)^2 \text{ sq. cm.}$$

$$\text{Resistance} = \frac{Sl}{A}$$

$$30 = \frac{0{\cdot}000048 \times l}{\pi \times (0{\cdot}011)^2}$$

$$\therefore\ l = \frac{30 \times 3{\cdot}14 \times (0{\cdot}011)^2}{0{\cdot}000048}$$

$$= 238 \text{ cm.}$$

Electromotive force.—In the consideration of potential difference and its relation to current, only those conductors have been taken into account which carry the current, but are incapable of supplying energy to the circuit. There would be no current in a circuit consisting solely of such conductors. For the maintenance of current it is necessary that there should be a supply of energy at some point or points of the circuit. This supply is generally due to an electric cell, or a dynamo. In the case of the cell, energy of chemical combination is converted into energy of current, to be

dissipated as heat in the various parts of the current circuit. In the dynamo, energy in the mechanical form, supplied by some form of engine, is converted into energy of electric current. Wherever energy of some other form is converted into energy of electric current there is said to be an **electromotive force,** commonly written **e.m.f.**

Electromotive force is measured in the same units as potential difference; that is, in terms of the rate of working for unit current. **The absolute unit of e.m.f. is acting when 1 erg per second is the rate of using energy for the maintenance of 1 absolute unit of current in the circuit.** Similarly, an electromotive force of 1 volt maintaining a current of one ampere supplies energy to the circuit at the rate of 10^7 ergs per second or 1 watt.

Complete circuit.—A current circuit presents many resemblances to a system of pipes in which water is caused to circulate by means of a pump. There is a drop of pressure from point to point round the circuit, and in the pump the pressure is brought up again, the increase in pressure produced by the pump being the sum of all the drops in pressure on travelling round the circuit. In a similar way there is a fall of potential all round the circuit, which is made up at the source of electromotive force. The electromotive force is equal to the sum of the falls of potential on travelling round the circuit, and obeys the same laws.

Thus, $$\frac{\textbf{electromotive force}}{\textbf{current}} = \textbf{total resistance of circuit}$$

For example, consider a circuit made up of a cell of e.m.f. 1·6 volt and external resistance of 3 ohms, the resistance of the cell itself being 0·2 ohm.

Total resistance in circuit $= 3 + 0{\cdot}2 = 3{\cdot}2$ ohms

$$\therefore \text{ Current} = \frac{\text{e.m.f.}}{\text{total resistance}} = \frac{1{\cdot}6}{3{\cdot}2}$$

$$= 0{\cdot}5 \text{ ampere}$$

It must be noticed that the source of electromotive force, either cell or battery, is itself a conductor, and like all conductors it has resistance, and this resistance must be included in finding the total resistance of the circuit.

In the case of more than one cell in series, each cell contributes energy to the circuit at a rate depending upon its own electromotive force. It is therefore necessary to **add**

the electromotive forces of the separate cells in order to obtain the effective electromotive force in the circuit. If, however, the cells are in parallel, the current divides between them and their electromotive forces are not added. When the electromotive forces of the cells in parallel are not equal there is no simple method of calculating the current in the circuit, but if they are equal, the effective electromotive force in the circuit is that of a single cell.

These points will be illustrated by means of a few examples.

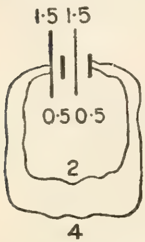


FIG. 68.—Problem.

Example (i).—A battery of two cells in series, each having e.m.f. 1·5 volt and internal resistance 0·5 ohm, supplies current to two wires of 2 and 4 ohms in parallel. What is the current in the cells?

$$\text{Total e.m.f.} = 3{\cdot}0 \text{ volts}$$

$$\text{Resistance of wires} = \text{R ohms}$$

$$\text{where,}\quad \frac{1}{R} = \frac{1}{2} + \frac{1}{4} = \frac{3}{4}$$

$$\therefore\ R = \tfrac{4}{3} \text{ ohm}$$

$$\text{Resistance of battery} = 1 \text{ ohm}$$

$$\therefore\ \text{Total resistance} = \tfrac{4}{3} + 1 = \tfrac{7}{3} \text{ ohms}$$

$$\therefore\ \text{Current} = \frac{3}{\tfrac{7}{3}} = \frac{9}{7} = 1{\cdot}28 \text{ ampere}$$

Example (ii).—Fifteen incandescent lamps in parallel are lighted by a dynamo giving an e.m.f. of 240 volts. If the resistance of each lamp is 90 ohms and that of the dynamo is negligible, find the total current and the horse-power of the dynamo.

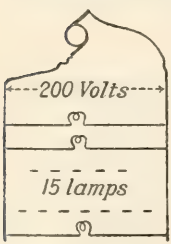


FIG 69.—Problem.

The arrangement is shown in Fig. 69. Since the lamps are in parallel, their combined resistance is given by—

$$\frac{1}{R} = \frac{1}{90} + \frac{1}{90} + \ldots \text{ to 15 terms}$$

$$= \tfrac{15}{90}$$

$$\therefore\ R = \tfrac{90}{15} = 6 \text{ ohms}$$

$$\therefore\ \text{Current} = \tfrac{240}{6}$$

$$= 40 \text{ amperes}$$

$$\text{Rate of working} = \text{e.m.f.} \times \text{current}$$
$$= 240 \times 40 \text{ watts}$$
$$= \frac{240 \times 40}{746} \text{ horse-power}$$
$$= 12{\cdot}9 \text{ horse-power}$$

Example (iii).—A circuit consists of a cell of e.m.f. 1·8 volt and internal resistance 0·6 ohm together with an external resistance of 3 ohms. Find the p.d. between the terminals of the cell

$$\text{Total resistance} = 3 + 0{\cdot}6 = 3{\cdot}6 \text{ ohms}$$
$$\therefore \text{ Current} = \frac{1{\cdot}8}{3{\cdot}6} = 0{\cdot}5 \text{ ampere}$$

The terminals of the cell are also the ends of the conductor,

$$\therefore \text{ p.d. required} = \text{current} \times \text{resistance}$$
$$= 0{\cdot}5 \times 3$$
$$= 1{\cdot}5 \text{ volt}$$

This method is simpler than to multiply the resistance of the cell by the current, as the source of e.m.f. lies between the cell terminals and must be taken into account. Thus—

$$\left.\begin{array}{r}\text{potential drop}\\ \text{in cell}\end{array}\right\} = \text{current} \times \text{resistance}$$
$$= 0{\cdot}5 \times 0{\cdot}6$$
$$= 0{\cdot}3 \text{ volt}$$
$$\therefore \left.\begin{array}{r}\text{p.d. between}\\ \text{terminals}\\ \text{of cell}\end{array}\right\} = 1{\cdot}8 - 0{\cdot}3$$
$$= 1{\cdot}5 \text{ volt}$$

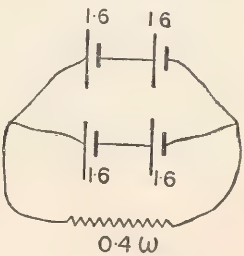


FIG. 70.—Problem.

Example (iv).—Four cells arranged as in Fig. 70 maintain a current in a resistance of 0·4 ohm. The e.m.f. of each cell is 1·6 volt and its resistance 0·6 ohm. What is the current in the 0·4 ohm conductor?

$$\text{Resistance of each battery branch} = 1{\cdot}2 \text{ ohm}$$
$$\therefore \frac{1}{\text{effective resistance of battery}} = \frac{1}{1{\cdot}2} + \frac{1}{1{\cdot}2} = \frac{2}{1{\cdot}2} = \frac{1}{0{\cdot}6}$$
$$\text{Effective resistance of battery} = 0{\cdot}6 \text{ ohm}$$
$$\text{Total resistance of circuit} = 0{\cdot}6 + 0{\cdot}4$$
$$= 1 \text{ ohm}$$
$$\text{Effective e.m.f. in circuit} = 1{\cdot}6 \times 2 = 3{\cdot}2 \text{ volts}$$
$$\therefore \text{ Current} = \frac{3{\cdot}2}{1}$$
$$= 3{\cdot}2 \text{ amperes}$$

Heat produced in conductors.—In the general case in which the current is flowing in a neutral conductor which does not affect the electromotive force in the circuit, the work done in a second in maintaining the current is—

$$\text{Current} \times \text{p.d.} = i \,.\, e = i^2 r = \frac{e^2}{r}$$

This work is converted into heat within the conductor and plays a similar part to the heat produced when friction resists the motion of one body over another. It is well known that when $4{\cdot}18 \times 10^7$ ergs are converted into heat, one calorie is produced; that is, the heat produced is sufficient to raise one gramme of water one degree centigrade in temperature.

From this it follows that the rate of working when a current is maintained in a conductor may be measured either in ergs per second or in calories per second.

$$\text{Thus rate of production of heat} = \frac{i \times e}{4{\cdot}18 \times 10^7} \text{ calories per second}$$

If the current and potential difference are measured in amperes and volts, the rate of working may still be expressed in calories per second, since $\mathrm{I} \times \mathrm{E} \times 10^7$ is the rate of working in ergs per second (p. 75).

$$\begin{aligned} \text{Rate of production of heat} &= \frac{\mathrm{I} \times \mathrm{E} \times 10^7}{4{\cdot}18 \times 10^7} \\ &= 0{\cdot}239\,\mathrm{IE} \\ &= 0{\cdot}239\,\mathrm{I}^2\mathrm{R} \\ &= \frac{0{\cdot}239\,\mathrm{E}^2}{\mathrm{R}} \text{ calories per second} \end{aligned}$$

Example.—A wire of 0·25 ohm resistance is immersed in 280 grammes of water and a current of 4·2 amperes is passed through it. If the temperature of the water at the beginning is 12·5° C., what is the temperature at the end of a quarter of an hour?

$$\begin{aligned} \text{Rate of production of heat} &= 0{\cdot}239 \times 4{\cdot}2^2 \times 0{\cdot}25 \text{ calories per second} \\ \text{Total heat produced} &= 0{\cdot}239 \times 4{\cdot}2^2 \times 0{\cdot}25 \times 900 \text{ calories} \\ \text{But heat produced} &= \text{mass of water} \times \text{rise in temperature} \\ &= 280 \times t \\ \therefore\ t &= \frac{0{\cdot}239 \times 4{\cdot}2^2 \times 0{\cdot}25 \times 900}{280} \\ &= 3{\cdot}39 \text{ degrees} \\ \therefore\ \text{Final temperature} &= 12{\cdot}5 + 3{\cdot}39 \\ &= 15{\cdot}89^\circ\ \text{C.} \end{aligned}$$

Utilisation of heat.—The heat produced by the electric

current is employed for many purposes. In the case of the incandescent lamp (p. 2) the heat is sufficient to produce a high enough temperature for the emission of light by a metallic filament, usually of tungsten. Where larger currents, with smaller rises of temperature but large production of heat, are required, as in the cases of electric kettles, irons and radiators, metallic wires are used, which are sometimes embedded in porcelain. The most useful wires for this purpose consist of various alloys of nickel and chromium.

Copper wires and fuses.—Another useful device consists in placing a piece of copper wire in the circuit to guard against excess of current. If the current should, through any accidental circumstances such as short-circuiting, rise to a dangerous value, the heat produced in the copper wire fuses it and so breaks the circuit. Such safety devices are called **fuses**. The surface of the copper wire is usually tinned to prevent oxidation of the copper, and when the fuse is used in a circuit carrying small current, pure tin wire is used, as this does not oxidise. Also the tin wire is much thicker than the copper wire which would fuse at the same temperature, both on account of the higher resistivity, and of the lower melting-point of tin.

TABLE OF COPPER WIRES.

Standard wire gauge.	Diameter in mm.	Ohms per metre.	Fusing current in amperes.
10	3·251	0·0020	—
12	2·642	0·0031	344
14	2·032	0·0052	232
16	1·626	0·0082	166
18	1·219	0·0145	108
20	0·9144	0·0258	70
22	0·7112	0·0435	48
24	0·5588	0·070	33
26	0·4572	0·105	26
28	0·3759	0·155	18
30	0·3149	0·222	14
32	0·2743	0·293	11·5
34	0·2337	0·404	9·0
36	0·1930	0·590	6·8
38	0·1524	0·950	4·8
40	0·1219	1·48	3·41
42	0·1016	2·10	2·59
44	0·0813	3·30	1·85
46	0·0610	5·90	1·25

Exercises on Chapter VI

1. What is the absolute unit of potential difference? Find the amount of work performed in 3 minutes when the current in a conductor is 1·6 and the p.d. between its ends 84 absolute units.

2. Give the relation of the ampere to the absolute unit of current, and the volt to the absolute unit of p.d. Find the work in ergs per second when the current in a conductor is 0·62 ampere and the p.d. between its ends is 15 volts.

3. State Ohm's law and indicate how it may be proved.

4. Define the "resistance" of a conductor and deduce the value of the ohm in terms of absolute units of resistance.

5. Calculate the combined resistances of four conductors of resistances 8, 10, 12 and 14 ohms respectively in parallel. If the total current is 1·3 ampere, find the current in the 12 ohm conductor.

6. What is specific resistance or resistivity? Find the resistance of a wire of length 24 metres and diameter 0·4 millimetre if the resistivity of the material is 0·000042.

7. Explain the difference between electromotive force and potential difference.

Find the potential difference between the terminals of a cell of e.m.f. 1·5 volts and internal resistance 2·5 ohms, when the terminals are joined by a wire of resistance 5·5 ohms.

8. Two cells each of e.m.f. 2·1 volts and of negligible resistance, are in series and maintain a current in a circuit which includes a conductor of resistance 1·5 ohm. What must be the resistance of the rest of the circuit if the p.d. between the ends of the conductor is 2 volts?

9. A dynamo maintains 200 incandescent lamps alight. If the lamps are in parallel and each has a resistance of 80 ohms and requires a p.d. of 160 volts, what is the e.m.f. of the dynamo if its internal resistance is 0·05 ohm?

10. Four cells each of e.m.f. 1·2 volt and internal resistance 0·4 ohm, produce a current in a conductor of resistance 1·5 ohm. Find the current (*a*) when the cells are in series and (*b*) when the cells are in parallel.

11. In the last question find the currents (*a*) and (*b*) when the external resistance is 120 ohms.

12. A circuit consists of a cell of e.m.f. 1·8 volt, and internal resistance 0·4 ohm, and an external resistance of 2·6 ohms. Find the calories produced per minute by the current in the external resistance and in the cell.

13. It is required to construct a heating coil which will produce 900 calories per minute on a supply circuit of 100 volts. What length of wire is required if its resistance is 3 ohms per metre?

14. Find the potential drop per metre in a copper wire of S.W.G. 22 when carrying a current of 3·5 amperes.

15. A coil whose resistance is intended to be 20 ohms is found actually to be 20·2 ohms. What resistance in parallel with it will make the combined resistance 20 ohms?

16. Two cells each of e.m.f. 1·6 volt and internal resistance 0·8 ohm

are used to maintain current in a conductor of resistance 5 ohms. Find the heat produced per minute in one cell when the cells are (*a*) in series, (*b*) in parallel.

17. A certain resistance box will only carry a current safely when the rate of working does not exceed 1 watt per 10 ohms. Find the greatest current allowable.

18. Find the horse-power required for an engine to drive a dynamo for lighting 70 incandescent lamps at 40 watts each, if 25 per cent. of the energy produced by the engine is wasted.

19. An arc lamp is run on a 100-volt supply, and the back e.m.f. in the arc itself is 40 volts. What resistance must be used in series with the arc if the current is to be 15 amperes?

20. Four arc lamps each giving a back e.m.f. of 40 volts are to be run on a 200-volt supply circuit, and the current in each lamp is to be 12 amperes. What is the most economical arrangement of lamps and auxiliary resistance?

CHAPTER VII

MEASUREMENTS

Current.—The measurement of electric current by means of the tangent galvanometer has been described in Chapter III, as well as the detection and comparison of currents by means of the more delicate forms of suspended needle galvanometer. Of late years, galvanometers in which the coil is suspended and the magnet fixed have come into very common use. A description of the principle of the suspended coil galvanometer will be deferred to Chapter IX, but it may be understood, on general dynamical principles, that if a coil carrying a current exerts a couple upon a magnet, the magnet will exert an equal and opposite couple upon the coil. This change in arrangement requires a redesigning of the galvanometer, for the magnet being fixed may be as heavy as we please, while the coil, being suspended, must now be as light as possible (p. 135). The suspended coil galvanometer has several advantages over the suspended magnet galvanometer, one of the chief being that it is not disturbed by changes in the external magnetic fields, such as those due to electric trams or the movement of neighbouring magnets.

Ammeters.—For the measurement of comparatively large currents with facility, it is desirable to have a fixed scale attached to the instrument, such that the deflection indicates at once the current flowing. Such an instrument is called an amperemeter or **ammeter.** The scale may vary in range from micro-amperes or millionths of an ampere for very small currents, milli-amperes or thousandths of an ampere for fairly small currents, to amperes or thousands of amperes for very large currents.

A typical form of the arrangement of an ammeter is shown

diagrammatically in Fig. 71. N and S are the poles of a permanent magnet and the coil C is pivoted top and bottom so that it can rotate between the poles. The current is brought to the coil by the delicate spring D which also serves as a control for the coil. The current may be taken out by a similar spring or other light conductor underneath. The pointer E and scale F enable the deflection to be observed.

Owing to the delicacy of the coil and leading spring, only very small currents pass through the coil. When greater currents are to be measured, they pass through a stout conductor S placed in parallel with the coil. This conductor is known as a **shunt**, and most of the current passes through it, only a small fraction passing through the coil. This arrangement enables the instrument maker to construct the moving parts of all the instruments alike. He then chooses a suitable shunt for each range of current required for the instrument.

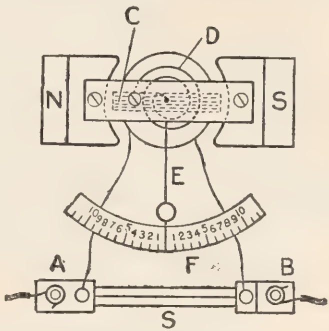


FIG. 71.—Moving coil ammeter.

Example.—The coil of an ammeter has a resistance of 5 ohms and gives one scale division for one milliampere. What is the resistance of the shunt if the current is to produce (*a*) 1 scale division per $\frac{1}{10}$ ampere, and (*b*) 1 scale division per ampere?

(*a*) $\frac{1}{10}$ ampere main current must correspond to $\frac{1}{1000}$ ampere in the coil.

Let S be the resistance of the shunt,

Then, $$\frac{1}{R}=\frac{1}{5}+\frac{1}{S}, \text{ and, } R=\frac{5S}{5+S}$$

$$\text{Current in coil} = \text{main current} \times \frac{5S}{5+S} \times \frac{1}{5} \text{ (p. 78)}$$

$$\frac{1}{1000}=\frac{1}{10}\cdot\frac{S}{5+S}$$

$$1000\ S = 50 + 10\ S$$

$$\therefore S=\frac{50}{990}$$

$$= 0{\cdot}0505 \text{ ohm}$$

D

(*b*) 1 ampere main current must correspond to $\frac{1}{1000}$ ampere in the coil.

$$\frac{1}{1000} = 1 \times \frac{S}{5+S}$$

$$1000S = 5+S$$

$$S = \frac{5}{999}$$

$$= 0{\cdot}00501 \text{ ohm}$$

A form of milli-ammeter of the suspended coil type, in which only one pivot is used for the suspension, is shown in Fig. 72, the instrument being opened for inspection.

There are several other types of ammeter in common use. One of these is the **hot-wire ammeter,** in which a current

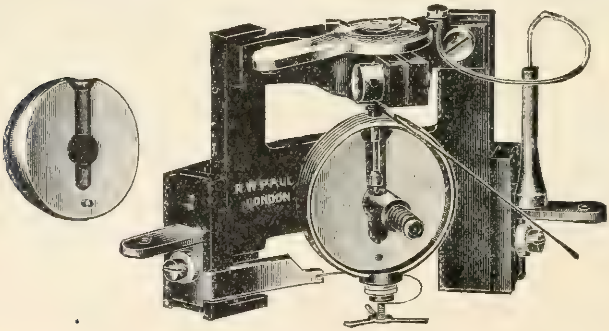


FIG. 72.—Moving coil milli-ammeter.

passing in a fine wire AB (Fig. 73) heats it and causes expansion. The amount of expansion is measured by the sag in the wire being taken up by a second wire CD, kept taut by the silk fibre EF and spring S. The silk fibre passes over the axle G of the pointer P. As before, the main current passes chiefly through a shunt.

Yet another form is the **soft-iron** ammeter, the principle of which is shown in Fig. 74. The current in the circular coil magnetises two soft-iron rods A and B which are parallel to the axis of the coil, producing two N poles at one end and two S poles at the other. Thus whatever the direction of the current there is a repulsion between the two soft-iron bars,

the force depending upon the strength of the current. A is fixed, but B is attached to a framework pivoted at the axis O. The attached pointer moves over the scale S, which is graduated to read the current. The weight of the moving parts brings the pointer to zero of the scale when no current is flowing, and serves as the control of the instrument.

Owing to the fact that an ammeter is placed in the circuit in which the current to be measured is flowing, **the resistance of the ammeter must always be very small.** This is to prevent any alteration of the current on introducing the ammeter and to avoid excessive heating in the instrument when the current is great.

Voltmeters.—For measuring potential difference, a modi-

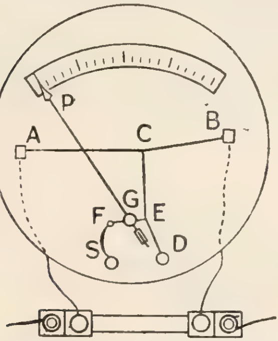


FIG. 73.—Hot-wire ammeter.

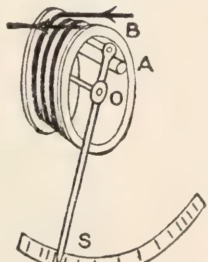


FIG. 74.—Soft-iron ammeter.

fied type of quadrant electrometer (p. 66) may be used ; but it is generally more convenient to use an instrument of the galvanometer type. Whenever the scale is graduated to give volts the instrument is called a **voltmeter.** Since a voltmeter is required to give the potential difference between two points of a circuit, it must be connected to these points ; that is, it is in parallel with the part of the circuit for which the fall of potential is required. For this reason **the resistance of a voltmeter must always be very high.** If this were not the case it would take an appreciable current and so disturb the circuit. Also when the p.d. between its ends is considerable it might take a great current and so be destroyed by overheating.

It is usual to use the same suspended-coil type of galvanometer for the voltmeter as for the ammeter, but in

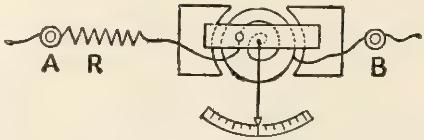


FIG. 75.—Voltmeter.

this case a suitable high resistance R (Fig. 75) is placed in **series** with the coil. Thus the resistance between A and B, the terminals of the voltmeter, is always very great.

Example.—If the coil of the instrument has a resistance of 5 ohms and a current of 1 milli-ampere gives a deflection of 1 scale division, what resistance must be used in series with it for the p.d. between its terminals to produce (*a*) 1 scale division per $\frac{1}{10}$ volt, and (*b*) 1 scale division per 10 volts ?

(*a*) $\frac{1}{10}$ volt is the p.d. for 1 milli-ampere or $\frac{1}{1000}$ ampere

$$\therefore \quad \frac{1}{10}=\frac{1}{1000}\times R, \text{ or } R = 100 \text{ ohms.}$$

But the resistance of the coil of the instrument is 5 ohms.

$\therefore$ **resistance placed in series with coil must be 95 ohms.**

(*b*) 10 volts is the p.d. for 1 milli-ampere, or $\frac{1}{1000}$ ampere

$$\therefore 10=\frac{1}{1000}\times R, \text{ or } R = 10000 \text{ ohms.}$$

But the resistance of the coil of the instrument is 5 ohms.

$\therefore$ **resistance placed in series with coil must be 9995 ohms.**

Resistance.—The measurement of the resistance of a conductor generally implies its comparison with some standard. These standards are of many types. The very low resistances used as shunts are illustrated in Fig. 76. Owing to the high values of current they may be required to carry they are made of fairly thick sheets of metal. When the standard resistance has a much higher value and is not required to carry large currents it consists of wire cut off to suitable length and wound on an insulating bobbin as shown in Fig. 77. The ends of the wire are soldered into massive brass blocks fixed on a sheet of ebonite. Such coils may be mounted in sets, as shown in Fig. 78. The brass plugs are ground carefully to fit the conical holes between the blocks.

Then the introduction of the plug short-circuits any given coil and is effective in removing that resistance from the circuit.

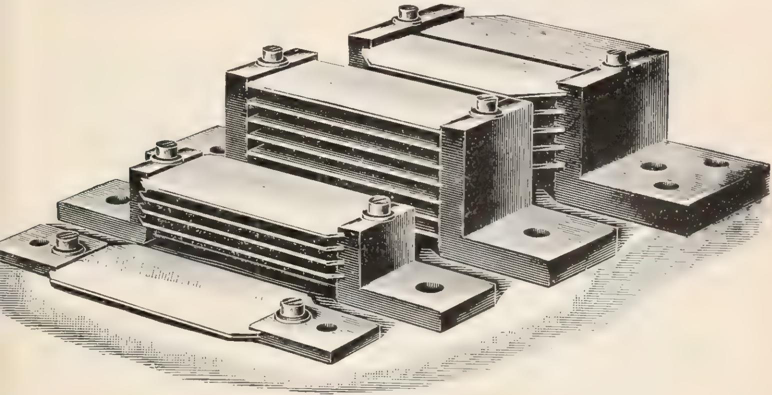


FIG. 76.

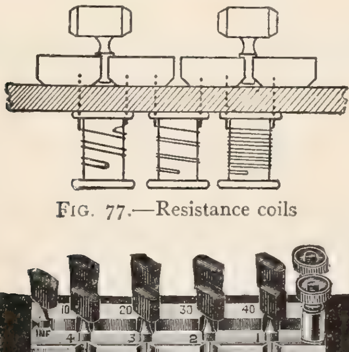


FIG. 77.—Resistance coils

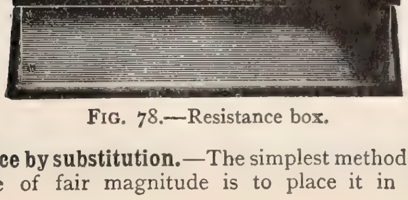


FIG. 78.—Resistance box.

Resistance by substitution.—The simplest method of measuring a resistance of fair magnitude is to place it in series with a

galvanometer G and cell C as in Fig. 79. The deflection on the galvanometer is then noted and the resistance R is removed from the circuit and an adjustable box of resistances put in its place. This change may be effected rapidly by changing the plug A to the position B. The resistance of the box is now varied until the same galvanometer deflection is obtained as when R was in the circuit. It follows then that the current is the same as before, since the e.m.f. is unchanged, the total resistance in the circuit is unaltered. The resistance R removed is therefore the resistance in the standard box introduced.

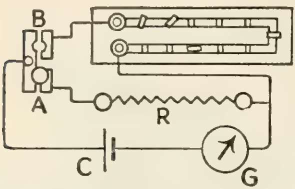


FIG. 79.—Resistance by substitution.

If the resistance to be measured is very high and a sensitive galvanometer is used, the deflection θ may be considered to be proportional to the current flowing.

Then, with unknown resistance R_1 in circuit—

$$\text{current} = \frac{E}{R_1} \propto \theta_1$$

With a high resistance standard R_2 in place of the unknown—

$$\text{current} = \frac{E}{R_2} \propto \theta_2$$

$$\therefore \frac{R_1}{R_2} = \frac{\theta_2}{\theta_1}$$

This method is satisfactory if R_1 and R_2 are of the order of a million ohms, that is one megohm, because the resistances of the cell and galvanometer are such a small part of the whole resistance of the circuit that they need not be taken into account.

Resistance of galvanometer.—The resistances of galvanometers vary from a small fraction of an ohm for simple galvanometers, and for those of the tangent form, to many thousands of ohms in other cases. There is only one good way of measuring the resistance of a galvanometer, which is to clamp the moving part and to treat the instrument as an ordinary conductor, measuring its resistance by the method on p. 98.

If the galvanometer is not very sensitive, its resistance may be found approximately by placing it in circuit with a cell of e.m.f.

E volt and an adjustable resistance box. The resistance R_1 in the box is found for some reasonable deflection; then the current in the galvanometer—

$$I_1 = \frac{E}{G+R_1}$$

where G is the resistance of the galvanometer, and the cell is supposed to have a negligible resistance. The resistance in the box is now changed to R_2 and the current changes to I_2, where—

$$I_2 = \frac{E}{G+R_2}$$

$$\therefore \quad \frac{I_1}{I_2} = \frac{G+R_2}{G+R_1} \qquad G = \frac{R_2 I_2 - R_1 I_1}{I_1 - I_2}$$

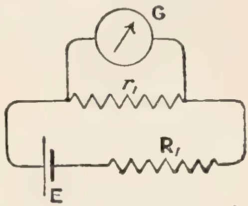


FIG. 80.—Resistance of galvanometer.

If the deflection is proportional to the current, θ_1 and θ_2 may be substituted for I_1 and I_2; but if the instrument is a tangent galvanometer tan θ_1 and tan θ_2 must be substituted for I_1 and I_2.

A rather more satisfactory method than the above is to use two adjustable resistance boxes r_1 and R_1 as in Fig. 80. R_1 and r_1 are varied until the galvanometer deflection has a reasonable value and the values are then noted. Then—

$$\text{Current in galvanometer} = \frac{E}{R_1 + \dfrac{r_1 G}{r_1+G}} \cdot \frac{r_1}{r_1+G}$$

The value of r_1 is now changed to r_2 and R_1 is varied until some new value R_2 is found, which causes the same deflection and therefore the same current in the galvanometer as before. Then—

$$\frac{E}{R_1 + \dfrac{r_1 G}{r_1+G}} \cdot \frac{r_1}{r_1+G} = \frac{E}{R_2 + \dfrac{r_2 G}{r_2+G}} \cdot \frac{r_2}{r_2+G}$$

$$\frac{r_1}{R_1 r_1 + R_1 G + r_1 G} = \frac{r_2}{R_2 r_2 + R_2 G + r_2 G}$$

$$G = \frac{r_1 r_2 (R_1 - R_2)}{r_1 R_2 - r_2 R_1}$$

Resistance by ammeter and voltmeter.—When the conductor is stout enough to carry a fairly large current

without injury, the direct method of measuring its resistance may be used. That is, the current flowing in it is measured by means of an ammeter A (Fig. 81), and the p.d. between the ends of the conductor by means of the voltmeter V. Dividing the p.d. by the current, the resistance is obtained. If an incandescent lamp be used as the conductor, the current may be varied by the rheostat R, and the value of the resistance of the lamp when cold and when hot can be found. On multiplying the current by the p.d. the watts used in the lamp are found, and on plotting watts against the resistance in the form of a graph, an interesting curve is obtained.

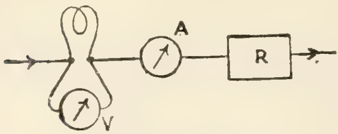


FIG. 81.—Resistance by ammeter and voltmeter.

If a photometer is available the candle-power of the lamp may be determined and the watts per candle-power can then be found.

Wheatstone's bridge.—An extremely delicate method for comparing resistances is due to Wheatstone and is known as the **Wheatstone's bridge**. It has very wide applicability. Consider four conductors, whose resistances are P, Q, R and S ohms respectively, placed as shown in Fig 82. The main current I divides between the two branches CAD and CBD, the current in the former being $I \cdot \frac{R+S}{P+Q+R+S}$ and that in CBD, $I \cdot \frac{P+Q}{P+Q+R+S}$.

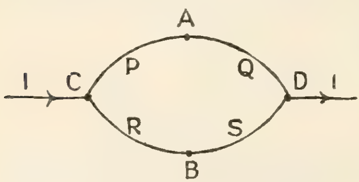


FIG. 82.—Divided circuit for Wheatstone's bridge.

Therefore—

$$\text{p.d. between C and A} = I \cdot \frac{(R+S)}{P+Q+R+S} \cdot P$$

$$\text{and p.d. between C and B} = I \cdot \frac{(P+Q)}{P+Q+R+S} \cdot R$$

If these are equal—

$$P(R+S) = R(P+Q)$$
$$PR+PS = PR+RQ$$

or,
$$\frac{P}{Q}=\frac{R}{S}$$

Hence, when the points A and B are at the same potential, the four resistances of the Wheatstone's bridge form a proportion. This condition is independent of the value of the current, and may be detected by the fact that if A and B are connected through a galvanometer, there will be no current and therefore no deflection. In Fig. 83 (i) the arrangement of the conductors is more clearly shown. If then one, say

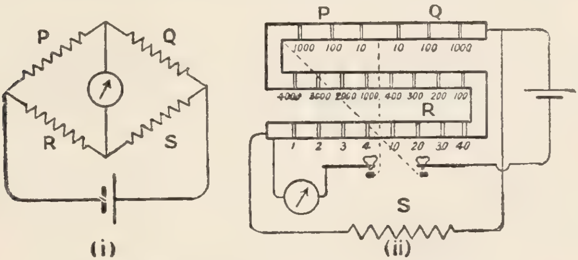


FIG. 83.—Wheatstone's bridge and Post-office box.

P, is unknown in value, the other resistances can be adjusted until the bridge balance is found, and the value of P can then be calculated.

When making measurements with the Wheatstone's bridge it is not advisable to let the current flow for longer than necessary. A key is therefore included in the battery circuit, which on being pressed, makes contact and allows the current to flow. On the other hand it is necessary that the currents should be fully established before the galvanometer is connected to the bridge. It may happen that there is a momentary kick of the galvanometer needle on starting or stopping the current due to other causes than the resistances in the bridge. Consequently there must be a key in the galvanometer circuit, which **must always be closed after the closing of the battery key.** This ensures the currents becoming steady before the galvanometer is connected to the bridge.

Post-office box.—One of the most convenient forms of

the Wheatstone's bridge was made for work in the postal service many years ago, and is still called the **Post-office box.** The general arrangement is shown in Fig. 83 (ii). The three arms P, Q and R of the bridge are made up of coils in the box, which are of the type seen in Fig. 77. The unknown resistance, or resistance to be measured, is made the fourth arm S, and is connected as shown. Usually two keys are provided in the box, one for the battery and one for the galvanometer. A comparison with Fig. 83 (i) will show how the connections are to be made. The student must remember, however, that different boxes are of different designs, but the cross connections inside the box are marked by the dotted lines as in the diagram, or the battery and galvanometer connections are directly indicated.

The coils in the Post-office box are so arranged that in the third arm R any resistance from 1 ohm to 11110 ohms can be introduced by unplugging the gaps suitably. P and Q are called the **ratio arms** and are chosen so that any decimal ratio from 1 : 100 to 100 : 1 may be used. This extends the range of usefulness of the Post-office box.

For example, suppose the resistance to be measured is of the order of 1000 ohms. The most suitable arrangement would then be P=1000 and Q=1000, and if the balance is obtained when 8462 ohms are introduced into the circuit R, this will be the resistance of S, which has therefore been found.

If, however, S is of the order of a few ohms, then the most suitable values are P=1000 and Q=10. Suppose then that the balance is perfect when R=846 ohms, the resistance of S is given

by—
$$\frac{1000}{10}=\frac{846}{S}$$

or,
$$S = 8{\cdot}46 \text{ ohms}$$

On the other hand, if the resistance of S is of the order of a million ohms or 1 megohm, the values to use are P=10, Q=1000, and if the balance is obtained when R=8462—

$$\frac{10}{1000}=\frac{8462}{S}$$

$$\therefore S = 846200 \text{ ohms}$$

The student should measure the resistance of several pieces of different kinds of wire, using as long a piece as is available. Then the length of each wire should be found, and also its cross section by measuring the diameter with a screw gauge.

The specific resistance or resistivity can then be found for each wire from the relation—

$$\text{Resistivity} = \frac{\text{Resistance} \times \text{area}}{\text{length}} \quad \text{(see p. 80)}$$

Metre bridge.—Another very common and convenient form of the Wheatstone's bridge is known as the **metre bridge.** Here two of the arms of the bridge are the parts of a fine wire 1 metre long. The arrangement is illustrated in Fig. 84. The metre wire RS is stretched over a scale divided in centimetres and millimetres. The ends of the wire are connected by thick copper strips of negligible resistance, gaps being left for conductors to be inserted at P and Q. The keys are not shown in Fig. 84. With this arrangement the

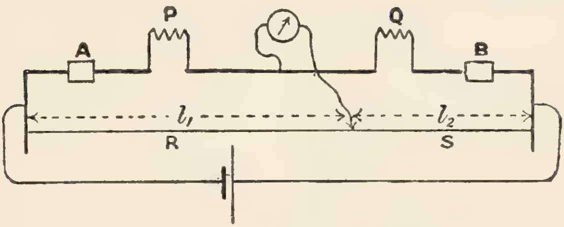


FIG. 84.—Diagram of metre bridge.

contact is moved along the wire until the galvanometer deflection is zero, when $\frac{P}{Q} = \frac{R}{S}$. But R and S are the segments of the uniform wire, so that if l_1 and l_2 are the lengths of the segments—

$$\frac{P}{Q} = \frac{l_1}{l_2}$$

The ratio of P to Q is then known and if either P or Q is a standard resistance, the resistance of the other can be calculated.

The actual form of a metre bridge is shown in Fig. 85. The contact with the wire is made through the key K, which slides in a brass V-guide connecting E and F, the terminals, to one of which the galvanometer or battery is connected. The gaps for the resistances which are to be compared are shown at P and Q. The gaps A and B may be occupied by

thick copper strips when not specially required. A glass window in the slider K has a fine scratch upon it which enables the position of the contact to be read on the metre scale. With a fairly sensitive galvanometer, the point of balance can be determined to within half a millimetre of the wire.

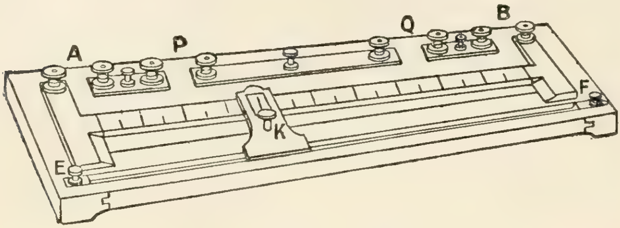


FIG. 85.—Metre bridge.

When the balance is near the middle of the wire the error of measurement is within half a millimetre in 50 centimetres, that is within 1 part in 1000. The balance should never be near one end of the wire, as the error is increased very much when one of the sections of the wire is very short.

If greater accuracy is required, the wire may be lengthened artificially by placing resistances in the gaps A and B. This involves a knowledge of the length of wire which has the same resistance as those placed in A or B, or, what is the same thing, it involves a knowledge of the resistance of the bridge wire in terms of those placed in A and B. Then, referring to Fig. 84, we have—

$$\frac{P}{Q}=\frac{A+\text{resistance of } l_1}{B+\text{resistance of } l_2}$$
$$=\frac{\text{Equivalent length of } A+l_1}{\text{Equivalent length of } B+l_2}$$

If A and B together have a resistance equal to, say, 99 times that of the bridge wire, the equivalent length of the bridge wire is increased from 1 metre to 100 metres. This increases the accuracy of measurement 100 times. It is, of course, necessary to choose the values of A and B so that the balance still comes on the actual wire of the bridge.

Carey-Foster's method.—A very good method of measuring the resistance of the bridge wire is due to Prof. Carey-Foster. Let two coils of resistances M and N be

placed in the gaps as shown in Fig. 86. Any two coils P and Q may be employed, provided that the balance comes

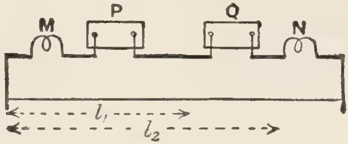


FIG. 86.—Carey-Foster bridge.

on the bridge wire. When the balance is found for length l_1

$$\frac{P}{Q}=\frac{M+l_1}{N+T-l_1}$$

where l_1 is now the resistance of the length l_1 of the bridge wire and T is the resistance of the whole wire.

Now interchange M and N and find a new balance when,

$$\frac{P}{Q}=\frac{N+l_2}{M+T-l_2}$$

$$\therefore \frac{M+l_1}{N+T-l_1}=\frac{N+l_2}{M+T-l_2}$$

On adding the denominators to the numerators we get—

$$\frac{M+N+T}{N+T-l_1}=\frac{M+N+T}{M+T-l_2}$$

$$\therefore N+T-l_1=M+T-l_2$$

or,
$$M-N=l_2-l_1$$

That is, the difference of resistance between M and N is equal to the resistance of a length l_2-l_1 of the bridge wire.

If N is a thick copper strip of zero resistance—

$$M=l_2-l_1$$

In this way the resistance per centimetre of the bridge wire may be found, and afterwards using two nearly equal coils as M and N their difference in resistance may be found as above.

Change of resistance with temperature.—The student is now in a position to investigate the variation of resistance with temperature for various wires. The wire itself should be wound in a coil

immersed in some non-conducting liquid such as paraffin oil for moderate temperatures, or glycerine for higher temperatures. Fairly thick copper wires, say No. 18, should be soldered to the ends of the coil to form connections to the Wheatstone's bridge, which may be a Post-office box or a metre bridge. A thermometer immersed in the oil, which should be kept well stirred, enables the temperature to be observed. The resistance is measured at ordinary temperatures and then at intervals of temperature, taking care that the temperature is steady each time the resistance is measured. A table is made of temperatures and resistances, and the results should be plotted on squared paper as shown in Fig. 87. In the case of the pure metals the points lie very nearly

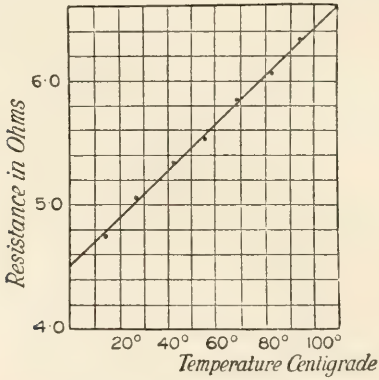


FIG. 87.—Graph of resistance and temperature.

on a straight line, if the range of temperature is not too great. A straight line should be drawn to lie evenly amongst the points and produced in both directions so that the resistances at 0° C. and at some other temperature, say 100° C., may be read from the curve.

The relation between resistance and temperature for moderate ranges of temperature is,

$$R_t = R_0(1 + \alpha t)$$

where R_t is the resistance at t° C. and R_0 that at 0° C. The quantity α is called **the coefficient of increase of resistance.** It may be found from the curve. Thus $R_0 = 4{\cdot}51$ ohms, $R_{100} = 6{\cdot}41$ ohms.

Then, $$6{\cdot}41 = 4{\cdot}51(1 + 100\alpha)$$

from which, $$\alpha = 0{\cdot}00421$$

Platinum resistance thermometer.—Platinum is a metal which is very little affected by the atmosphere, and retains its physical properties unchanged although subjected to considerable changes of temperature; that is, when its temperature is varied and brought back to its first value, the resistance will be found to have returned also to its original value. Also its temperature coefficient is very accurately known. It is thus a very suitable substance for using in the measurement of temperature by electrical resistance. Hence, if the resistance of a given platinum wire be known at several temperatures, it is possible by measuring its resistance, to calculate the temperature. It thus forms a

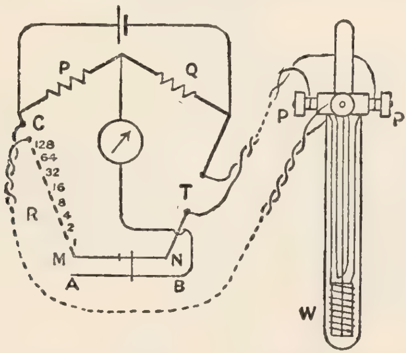


FIG. 88.—Callendar and Griffiths bridge.

very convenient and reliable thermometer, whose range is very great, far beyond that of any liquid thermometer, extending from the temperature of liquid oxygen (−183° C.) up to nearly the melting-point of platinum (1750° C.). The platinum resistance thermometer usually takes the form of a spiral of platinum wire W (Fig. 88) wound upon a mica frame and enclosed in a glass tube, if temperatures up to 300° C. are to be measured. For higher temperatures a porcelain tube is employed.

Any form of Wheatstone's bridge will serve for measuring the resistance of the platinum spiral. The tube is first immersed in melting ice (0° C.) and the resistance R_0 is measured. It is then immersed in the steam over boiling water at the normal atmospheric pressure (100° C.) and its resistance again measured,

giving R_{100}. If, then, the resistance at any unknown temperature R_t is measured, the temperature t can be calculated—

For,
$$R_{100}=R_0(1+100\alpha)$$
$$R_t=R_0(1+\alpha t)$$
$$\therefore\ R_{100}-R_0=R_0 100\alpha$$
$$R_t-R_0=R_0 t\alpha$$
$$\therefore\ t=100\frac{R_t-R_0}{R_{100}-R_0}$$

In a given case, if the resistance at 0° C. is 12·8 ohms and that at 100° C. 17·8 ohms—

$$t=100\frac{R_t-12\cdot8}{17\cdot8-12\cdot8}$$
$$=20(R_t-12\cdot8)$$

If then R_t is found to be 19·5 ohms

$$t=20(19\cdot5-12\cdot8)$$
$$=134^\circ \text{ C.}$$

It will be noticed that the interval between 0° C. and 100° C. corresponds to 5 ohms, and 0·05 ohm corresponds to 1° change. Since the change in resistance may be easily measured to within 0·001 ohm, this corresponds to $\frac{1}{50}$ degree.

Prof. Callendar found that the simple relation given above does not apply over extended ranges of temperature, in which case the equation is—

$$R_t=R_0(1+\alpha t+\beta t^2)$$

In order to find the second constant β, the resistance at a third temperature, usually that of boiling sulphur (444·53° C.), is employed. If, however, the temperature calculated from the simple platinum resistance formula be called t_p, the temperature t on the air thermometer scale can be found from the relation—

$$t-t_p=1\cdot5\left(\frac{t}{100}-1\right)\frac{t}{100}$$

A very convenient form of the Wheatstone's bridge for use with the platinum resistance thermometer has been devised by Callendar and Griffiths, and is shown diagramatically in Fig. 88. The ratio arms of the bridge P and Q are two equal coils. R is a set of coils for balancing, and the platinum thermometer is placed in the gap T. The fine balancing is done on the wire MN. In order to eliminate errors due to the resistance of the leads, a dummy pair of leads, lying alongside the actual pair, is placed in the gap C of the bridge. Since the two arms of the bridge are

equal in resistance, the resistance of the dummy leads balances that of the actual leads at all temperatures.

Electromotive force.—There are several simple methods of comparing the electromotive forces of different cells. On making a simple circuit of a tangent galvanometer, a resistance box, and one of the cells, a deflection, say θ_1, is observed—

$$\frac{E_1}{R} = I_1 = K \tan \theta_1$$

On replacing the cell by the other with which it is to be compared, a second deflection θ_2 is observed—

$$\frac{E_2}{R} = I_2 = K \tan \theta_2$$

$$\therefore \frac{E_1}{E_2} = \frac{\tan \theta_1}{\tan \theta_2}$$

This method is unsatisfactory since the resistances of the two cells have been assumed to be the same. If, however, a delicate reflecting galvanometer had been used, the resistance included in the circuit to keep the deflection down to a reasonable amount is of the nature of hundreds of thousands of ohms, and the resistances of the cells may then be neglected, and—

$$\frac{E_1}{E_2} = \frac{\theta_1}{\theta_2}$$

Sum and difference method.—Another method of getting over the difficulty introduced by the cells having different resistances is to use the tangent galvanometer still, having the cells both in the circuit at the same time, but first helping each other—

$$\frac{E_1 + E_2}{R} = K \tan \theta_1$$

then, opposing each other—

$$\frac{E_1 - E_2}{R} = K \tan \theta_2$$

$$\frac{E_1 + E_2}{E_1 - E_2} = \frac{\tan \theta_1}{\tan \theta_2}$$

$$\therefore \frac{E_1}{E_2} = \frac{\tan \theta_1 + \tan \theta_2}{\tan \theta_1 - \tan \theta_2}$$

Potentiometer.—The potentiometer affords by far the

most efficient and accurate means of comparing electromotive forces and potential differences. A straight wire carries a constant current, and if the wire is uniform there is consequently a uniform fall of potential from one end to the other. It is therefore possible to use this as a scale of potential difference and to balance the unknown e.m.f. or potential difference against it. If AB (Fig. 89) is the wire, a steady

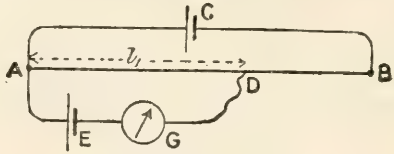


FIG. 89.—Potentiometer.

current is maintained in it by the cell C, which should be a secondary cell or accumulator. E is one of the cells whose e.m.f.'s are to be compared, and is joined up as shown. If its e.m.f. is equal to the potential difference between the points A and D on the wire, no current will flow in E, which fact is detected by the galvanometer G being undisturbed when the contact at D is made. Since the wire is uniform, the p.d. between A and D is proportional to the length l_1. If, then, E_1 is the e.m.f. of the cell—

$$E_1 = kl_1$$

On replacing the cell by the second cell of e.m.f. E_2, a second length of wire l_2 is found for which there is no deflection of the galvanometer. Then $E_2 = kl_2$

and, $$\frac{E_1}{E_2} = \frac{l_1}{l_2}$$

It should be noted that at the moment at which the balance is obtained, there is no current flowing in the cell E, and its resistance does not affect the point of balance. Also the method is very sensitive, as the wire may have any length desired, and the longer the wire the more open will be the scale of potential difference.

The student should compare the e.m.f.'s of a number of cells, using a simple potentiometer of the form shown in Fig. 90. A piece of platinoid or manganin wire is soldered to terminals A and B and stretched backwards and forwards

on a board, passing over screws at P, Q, R, S and T. If a piece of squared paper be pasted on the board before the

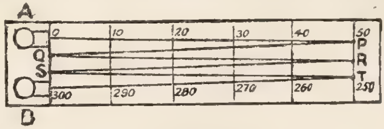


FIG. 90.—Simple potentiometer.

wire is fixed, it forms a very convenient scale on which the lengths l_1 and l_2 of the wire may be measured.

Measurement of current by the potentiometer.—Although the potentiometer is an instrument for comparing electromotive forces, yet, by the employment of a standard resistance, it may be used to measure current. If the current to be measured be passed through the standard resistance, the p.d. between the ends of the resistance may be found by comparison with the e.m.f. of a standard cell. The current can then be calculated.

Thus, if R (Fig. 91) is the value of the standard resistance in

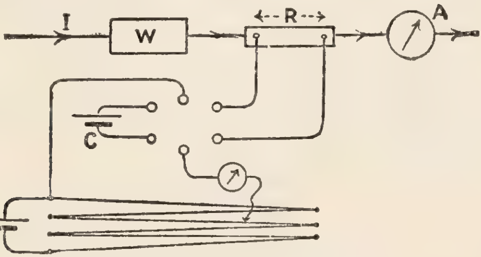


FIG. 91.—Current by potentiometer.

ohms, and the current I amperes flows in it, the p.d. between its ends is IR volts. With the arrangement in the diagram, the length l_1 of potentiometer wire is found, which gives zero deflection of the galvanometer. By means of the mercury key, the standard cell C is made to replace the resistance R, and the length l_2 of the potentiometer wire required for a balance is found. Then, if E is the e.m.f. of the cell C—

$$\frac{IR}{E}=\frac{l_1}{l_2}$$

$$\therefore\ I=\frac{E}{R}\cdot\frac{l_1}{l_2}$$

This method is of very wide application, for a suitable choice

of resistance will enable almost any current to be measured. If the current is about 1 ampere, a standard resistance of 1 ohm is suitable, as the p.d. over it will then be about 1 volt. If the current is 100 amperes, the standard resistance should be 0·01 ohm, and so on.

The calibration of an ammeter should be carried out by the student by this method. In Fig. 91 the ammeter A is placed so that the current in the standard resistance R flows through it. A variable rheostat W is also included in the circuit, so that the current may be varied and the true current corresponding to various ammeter readings may be found. A calibration curve of ammeter readings against current should be plotted. For exact measurements the cadmium cell (p. 124) should be used as the standard C, as its e.m.f. at various temperatures is known with great exactitude. If such great precision is not required a Daniell's cell may be used, the e.m.f. of which is about 1·07 volts.

Comparison of resistances by the potentiometer.—The experiment illustrated in Fig. 91 may be used for the comparison of resistances, and affords a method when the resistances are too small to be compared accurately by means of the Wheatstone's bridge. If R and W are the two resistances to be compared, let a steady current be passed through them as shown. Let the length of potentiometer wire l_1 corresponding to the p.d. IR, across R, be found. The wires to the potentiometer are then transferred from the resistance R to the ends of the resistance W, so that the length l_2 of potentiometer wire now required for a balance corresponds to the p.d. IW. Then—

$$\frac{IR}{IW} = \frac{l_1}{l_2}$$

$$\therefore \frac{R}{W} = \frac{l_1}{l_2}$$

The resistances compared by this method should not be of very different orders of magnitude, or a reasonable length of potentiometer for the high resistance will mean having a very small length for the low resistance. If the resistances are both very low a strong current may be used so that convenient lengths of potentiometer wire may be obtained.

Internal resistance of a cell.—That a cell has resistance has been seen several times. Being itself a conductor, it is no exception to the rule that it possesses resistance, that is, that there is a fall of potential when current flows in it, quite independently of the fact that the cell itself is the source of electromotive force. For, if the e.m.f. of the cell be

represented by OE in Fig. 92, and the resistance of the cell by OA, the resistance of the various parts of the external circuit

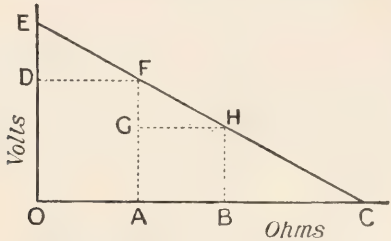


FIG. 92.—Resistance and p.d. diagram.

being represented respectively by AB and BC, then on joining E and C, the current in the circuit is represented by OE/OC, and the fall of potential over BC is $BC \times \frac{OE}{OC} = BC \times \frac{HB}{BC} = HB$. Similarly the p.d. between A and B is GF, and the fall of potential through the cell is ED.

The fall of potential through the cell is not equal to the difference of potential between its terminals because of the presence of the e.m.f. of the cell. The difference of potential between its terminals is best found by multiplying the current by the external resistance and is respreented by OD or AF in Fig. 92. (Also see p. 83.)

Measurement of internal resistance of a cell.—The potentiometer affords the best means of measuring the internal resistance of a cell. For, with the cell on **open circuit**, that is, with an infinite external resistance, the difference of potential between its terminals is equal to the e.m.f., E, of the cell. This may be seen by imagining OC to be made very great in Fig. 92. The cell E is connected to the potentiometer as shown in Fig. 93, and the length of potentiometer l_1 required for a balance is found.

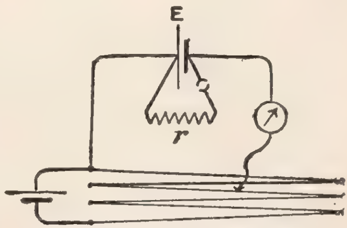


FIG. 93.—Internal resistance of cell.

The known resistance r is then connected across the cell,

whose internal resistance may be taken as R. The cell produces current $\frac{E}{R+r}$, and the p.d. between the ends of r is $\frac{Er}{R+r}$. The length of potentiometer required for the new balance (l_2) is then found, and—

$$\frac{E}{\frac{Er}{R+r}}=\frac{l_1}{l_2}$$

$$\frac{R+r}{r}=\frac{l_1}{l_2}$$

$$\frac{R}{r}=\frac{l_1-l_2}{l_2}$$

Since r is known, R can be calculated. The student should find the internal resistance of several cells in this way, in each case using various values for r. It will be found that the internal resistance of a cell depends upon the current which it is producing.

Zero error of potentiometer.—It is very difficult to make sure that the resistance of the wire of the potentiometer is proportional to the length as measured on the scale. There is generally some resistance in the soldering between the terminal and the beginning of the scale. A correction for this error may be found by taking three readings of length corresponding to three p.d.'s of which one is known to be the sum of the other two. Take two fairly high resistances PQ and QR (Fig. 94) of any values, which need not be known. On passing a steady current through the two resistances in series, we have—

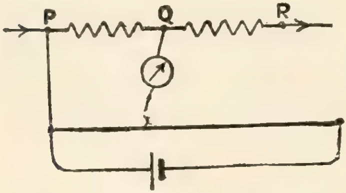


FIG. 94.—Zero error of potentiometer

p.d. between P and R=(p.d. between P and Q)+(p.d. between Q and R).

Let the lengths l_1 and l_2 be found on the potentiometer wire for the p.d.'s between P and Q, Q and R, and l_3 for the p.d. between P and R. Then, if there is no zero error of the instrument—

$$l_3=l_1+l_2$$

But if there is a zero error equal to a scale divisions, this must be added to each length and the true relation is—

$$(l_3+a)=(l_1+a)+(l_2+a)$$
$$\therefore\ a=l_3-l_1-l_2$$

The zero error a is thus found and must be applied in all future readings to obtain the true lengths which are proportional to the p.d.'s measured. The quantity a may, of course, be either positive or negative for any given potentiometer, or it may be so small as to be negligible.

Exercises on Chapter VII

1. Describe some form of ammeter which may be used for currents up to 10 amperes.

2. The coil of an ammeter has a resistance of 10 ohms, and a current of 1 milliampere in it produces a deflection of 3 scale divisions. What resistance of shunt must be used, so that the scale shall represent 1 ampere per division?

3. In the last question, what modification must be made, in order that the instrument shall become a voltmeter reading 1 volt per division of scale?

4. Describe a method of measuring resistance by the process of simple substitution.

5. How may the resistance of a galvanometer be measured, without employing a second galvanometer?

6. Give the theory of the Wheatstone's bridge.

7. Describe, giving diagram, the measurement of resistance by means of the Post-office box.

8. The ratio arms of a Post-office box are 1000 and 10, and a resistance of 136 is required in the third arm to produce a balance when 3 metres of wire of diameter 0·22 mm. form the fourth arm. What is the resistivity of the wire?

9. Explain how the resistance of the wire of a metre bridge may be measured.

10. Describe how the difference in resistance of two very nearly equal resistance coils may be measured by the Carey-Foster method.

11. Explain how the change in resistance of a platinum wire may be employed for the measurement of temperature.

12. If the resistance of a platinum wire is found to be 6·4 ohms at 0° C. and 8·9 ohms at 100° C., what will be the temperature when the resistance is 7·3 ohms?

13. Two cells, A and B, in series cause a deflection of 54° in the needle of a tangent galvanometer. When B is reversed the deflection is 4° in the same direction as before. Find the ratio of the e.m.f. of A to that of B.

14. Describe the potentiometer method of comparing the e.m.f.'s of cells.

15. A potentiometer is used for measuring current. The length of wire for a balance is 58·8 cm. for a cell of e.m.f. 1·1 volt, and 46·2 cm. for the p.d. over a resistance of 0·5 ohm carrying a current which produces a reading of 1·8 on an ammeter. What is the correction to be applied to the ammeter at this reading?

16. Explain the use of the potentiometer to measure the internal resistance of a cell.

17. The length of potentiometer wire required to produce a balance with a cell on open circuit is 105·6 cm., and when the cell terminals are connected by a resistance of 5 ohms the length of wire is 98·9 cm. What is the internal resistance of the cell, if the zero error of the potentiometer is +0·5 cm.?

18. A piece of wire forms the fourth arm of a Post-office box, and the ratio arms are 1000 : 10. With the wire at 15° C. the third arm is 1356 for a balance, and when the wire is at 95° C. the third arm is 1535. What is the temperature coefficient of resistance of the wire?

CHAPTER VIII

ELECTROLYSIS

Early discoveries.—The discoveries of Galvani and Volta led to our knowledge of the electric current, which for a long time was called a "Voltaic" current. Galvani found that a circuit of dissimilar metals could under suitable circumstances stimulate certain nerves, but he could not increase the effect. Volta found that by building up a "pile," or as we should now call it **a battery,** of layers of copper, acidulated water in cloth, and zinc, repeated until about 100 were in series, the effects discovered by Galvani were greatly accentuated. In this country Nicholson and Carlisle, in setting up the Voltaic pile, noticed that if the circuit were completed through a drop of water, bubbles of gas appeared at one terminal and the other became oxidised. At a later date Sir Humphrey Davy passed the current through fused soda and obtained a metallic bead to which the name sodium was given. In a similar manner the metal potassium was discovered. It was to Michael Faraday, however, that we owe the greatest step in our knowledge of these processes.

Faraday not only enunciated quantitatively the laws governing the passage of electric current through solutions, but suggested a nomenclature which is still in general use. The solution through which the current passes is called an **electrolyte,** and the term **electrolysis** denotes the chemical changes which accompany the passage of the current. The conductors by which the current enters and leaves the electrolyte are called **electrodes,** that by which the current enters being the **anode,** and that by which it leaves the **cathode.** The products of decomposition were called **ions,** but this term is now generally used in a somewhat different sense, as will be seen in Chapter XIII.

Many liquids are almost complete non-conductors of

electricity, but there are certain solutions which conduct comparatively well. These are solutions of certain salts or acids, and the passage of the current is accompanied by the liberation of **the metal at the cathode**, and the **acid radicle at the anode.** Whether these substances remain in their isolated state or whether they combine with other substances present depends, of course, upon their nature.

Electrolysis of water.—The case of the passage of a current through water is of great importance. Pure water is almost a non-conductor, but very small amounts of salt or acid in solution render it conducting. If a vessel such as that shown in Fig. 95 be filled with water containing a little sulphuric acid (H_2SO_4) in solution, and a current passed through the solution from one platinum electrode to the other, it will be found that bubbles of gas rise from the electrodes. If the electrodes are situated one in each tube, the gases are collected and can be measured. It will then be found that the gas at the cathode has about twice the volume of that at the anode. On applying the usual tests it is found that that from the cathode is hydrogen, while that from the anode is oxygen—the constituents of water.

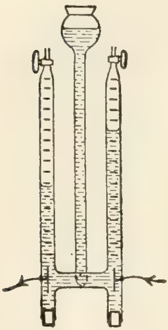


FIG. 95.—Electrolysis of water.

The process of electrolysis is most correctly represented by considering that the hydrogen of the sulphuric acid is liberated at the cathode, where it bubbles away. But at the anode it is the acid radicle SO_4 which is liberated. This substance, however, cannot exist alone, and immediately combines with the hydrogen of the water forming sulphuric acid and liberating the oxygen, which bubbles away,

$$H_2O + SO_4 = H_2SO_4 + O$$

It is therefore the sulphuric acid and not the water which is essential to the passage of the current, although the final result is that the water is decomposed while the sulphuric acid remains.

Faraday's laws of electrolysis.—As the result of measurements made on the passage of a current through various electrolytes Faraday enunciated the following two laws governing the amount of chemical action produced.

1st Law.—The amount of any electrolyte decomposed by the passage of an electric current is proportional to the quantity of electricity passing through the electrolyte.

Since the quantity of electricity passing is the product of the current and the time for which the current flows, the 1st law may be written—

The quantity of an electrolyte decomposed is proportional to the current and to the time for which the current flows.

2nd Law.—The amount of any substance liberated by electrolysis by a given quantity of electricity is proportional to the chemical equivalent of the substance.

In order to understand this law, consider a number of electrolytic cells in series as in Fig. 96. Then the same current

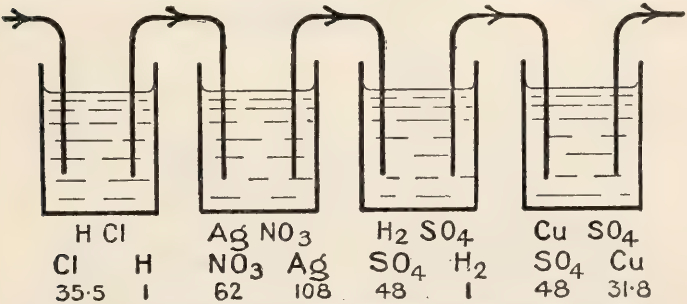


FIG. 96.—Electrolytic cells representing the laws of electrolysis.

must pass for the same time through all of them. In other words, the same quantity of electricity must pass through each cell.

Let the current be supposed to flow until 1 grm. of hydrogen is liberated in the first cell. Then the 2nd law indicates that 1 grm. of hydrogen will be liberated in the third cell, 35·5 grms. of chlorine in the first, 108 grms. of silver and 62 grms. of NO_3 in the second cell. Since the chemical equivalent of 2 grms. of hydrogen is 96 grms. of SO_4, it follows that when 1 grm. of hydrogen is liberated only 48 grms. of SO_4 will be liberated. This applies also to the fourth cell, in which, therefore, 31·8 grms. of copper will be liberated. Thus the chemical equivalents of the monovalent elements are their atomic weights, of the divalent elements half their atomic weights, and so on.

Electrochemical equivalents.—It follows from Faraday's

laws of electrolysis that if the amount of any one substance liberated by a given current in a given time be found by measurement, then the amount of any substance for any current in any time can be calculated if the atomic weights and valencies of the substances concerned are known.

The weight of any substance liberated by one ampere in one second is called its **electrochemical equivalent.**

The substance chosen for the standard measurement of an electrochemical equivalent was silver, because it has a very large value for its chemical equivalent. Further, it is a metal which can be deposited in a pure form in a hard deposit from a solution

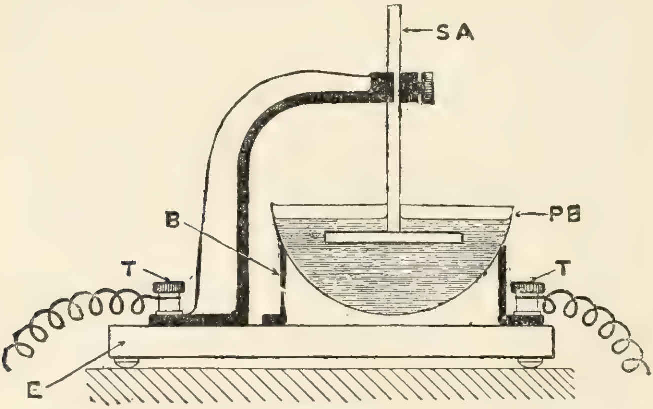


FIG. 97.—Silver voltameter.

of its salt, silver nitrate. Lord Rayleigh, in 1884, determined the electrochemical equivalent of silver, using an apparatus shown in Fig. 97. PB is a platinum basin which is first carefully cleaned and weighed, and into which a solution of silver nitrate is placed. The current is led in and out by the terminals T, and the basin is supported on a brass ring B. The platinum basin is made the cathode, so that metallic silver is deposited upon it. After passing the current for a known time, the basin is washed, dried, and weighed. The weight of silver deposited is therefore known, and from the relation—

weight of deposit = electrochemical equivalent × current × time

or, $w = z \, . \, \mathrm{I} \, . \, t$

the electrochemical equivalent is calculated. The anode consists

of a silver plate, so that the action of the NO_3 liberated is to form silver nitrate again, and the strength of the solution remains constant. It is necessary to be careful that the current is not too great, or the deposit will not be sufficiently hard to ensure that none of it is removed in washing. The current should not exceed 0·03 ampere per square centimetre of surface of the cathode. Such an apparatus is called a **voltameter,** in this case a silver voltameter.

By means of the silver voltameter Lord Rayleigh found the electrochemical equivalent of silver to be 0·0011180, but later determinations give the value 0·00111827.

TABLE OF ELECTROCHEMICAL EQUIVALENTS.

Substance.	Valency.	Atomic weight (O = 16).	Electrochemical equivalent.
Aluminium (Al) . .	3	27·1	0·0000936
Chlorine (Cl) . . .	1	35·46	0·0003676
Copper (Cu) . . .	1 or 2	63·57	0·0003293
Hydrogen (H) . . .	1	1·008	0·00001044
Oxygen (O)	2	16·0	0·00008293
Silver (Ag)	1	107·88	0·0011183
Sodium (Na) . . .	1	23·00	0·0002384
Zinc (Zn)	2	65·37	0·0003387

Copper voltameter.—For the measurement of current in the standardising of instruments such as galvanometers and ammeters, the voltameter is of very great use. The only apparatus of great precision required is a delicate balance. When a very high order of accuracy is required the silver voltameter must be used. But for calibrations in which an error of 1 in 1000 is allowable, the **copper voltameter** affords a suitable substitute.

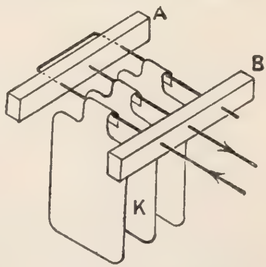


FIG. 98.—Copper voltameter.

The copper voltameter usually consists of three copper plates hanging from stout wires supported by wooden rods A and B (Fig. 98). The middle plate, K, is the cathode, but the two outer plates, which are connected together, constitute the anode. The plate K is first

cleaned by scrubbing with emery paper or sand, and is replaced in position. The plates are lowered into a solution of copper sulphate contained in a jar J (Fig. 99), the solution consisting of copper sulphate crystals dissolved in about four times their weight of water, with a few drops of concentrated sulphuric acid added. After connecting up a battery B, the voltameter, an adjustable rheostat, and the ammeter to be calibrated A, the current is adjusted to a convenient amount by means of the rheostat. The current should not exceed 1 ampere to 50 square centimetres of cathode surface, or the copper deposit will be soft and liable to be removed. After the current is adjusted, the cathode is removed, washed, dried and weighed, and on replacing it the time should be noted. After passing the current for half an hour to an hour, the longer the better, the cathode is removed and again washed, dried and weighed, the instant of removing it being recorded. Care must be taken that the current remains constant for the whole time that it is passing, the rheostat being gently adjusted, if necessary, to ensure this constancy. From the weight of deposit and the time for which the current flows, the current can be calculated, knowing the electrochemical equivalent of copper, and the error of the instrument is found.

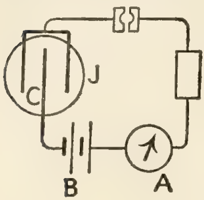


FIG. 99.—Calibration of ammeter.

EXAMPLE.—Reading of ammeter = 0·95 ampere
Wt. of cathode before deposit = 3·585 grms.
" " after " = 4·611 grms.
Time for which current passed = 55 minutes
Wt. of deposit = 1·026 grms.
Wt. of deposit = $I . z . t$
$1{\cdot}026 = I \times 0{\cdot}0003293 \times 55 \times 60$
$\therefore\ I = 0{\cdot}944$ ampere
and, Error of instrument at 0·95 = −0·006 ampere

Water voltameter.—The apparatus shown in Fig. 95 may be used as a voltameter. On passing the current for a known time, the volume of the hydrogen may be observed from the graduations on the vertical tube in which it is collected. The density of hydrogen at 0° C. and 76 cm. of mercury pressure is known to be 0·0896 grm. per litre, and the volume of the gas collected must therefore be reduced from its amount at the temperature and pressure in the tube to the standard temperature and pressure. Due allowance must, of course, be made for the water vapour present in the gas

as collected over water. In this way the mass of hydrogen liberated is found, and from its electrochemical equivalent (p. 117) the current can be calculated. This method is not nearly so accurate as that of the copper or the silver voltameter and is rarely used.

Theory of electrolysis.—Very early in the history of electrolysis it was recognised that the atom of hydrogen, or the metal, carried a charge of positive electricity, and the acid radicle a negative charge. For the atoms of the metal are driven in the direction of the electric field produced between the electrodes by the battery applied, and the acid radicle is driven in the opposite direction. Also, the second of Faraday's laws of electrolysis makes it clear that every monovalent atom carries the same amount of electricity, every divalent atom twice that amount, and so on. The movement of these atoms with their charges, positive towards the cathode, and negative towards the anode, constitutes the current through the electrolyte.

If, then, a current be passed through a solution of silver nitrate until 107·88 grms. of silver is liberated—

$$107{\cdot}88 = \mathrm{I} \times 0{\cdot}0011183 \times t$$

or,

$$\mathrm{I} \times t = \frac{107{\cdot}88}{0{\cdot}0011183} = 96470$$

Now, $\mathrm{I} \times t$, or (current in amperes) $\times$ (time in seconds) is the number of coulombs (p. 72) which passes.

Therefore the atoms in 107·88 grms. of silver altogether carry 96470 coulombs of positive electricity.

Similarly 23 grms. of sodium or 1·008 grms. of hydrogen would carry the same amount of positive electricity, while 35·46 grms. of chlorine would carry an equal amount of negative electricity. All these quantities contain the same number of atoms, and it follows that all monovalent atoms carry the same amount of electricity, also divalent atoms carry twice this amount, and so on.

Electrolytic dissociation.—It was thought, at one time, that the atoms constituting a molecule in the electrolyte were pulled apart by the applied electric field. Thus a considerable electromotive force would be necessary to start the process of electrolysis, as an insufficient electric field would not be able to cause separation of the atoms in combination. It is found, however, that Ohm's law applies to electrolytes, and the above view is

abandoned in favour of the theory of electrolytic dissociation. According to this theory some of the molecules of the substance in solution are resolved into their constituent atoms, which wander freely about the solution. As an example, consider the case of a simple salt such as sodium chloride (NaCl) dissolved in water. Some of the molecules are dissociated into a sodium atom with a positive charge of electricity ($\overset{+}{\text{Na}}$) and a chlorine atom with a negative charge ($\overset{-}{\text{Cl}}$). These atoms with their charges are called **ions**, positive ions and negative ions.

How the charges are acquired by the ions is not certain, but there are many reasons for believing that some atoms readily lose a fundamental small charge of negative electricity called an **electron** (p. 190) and would therefore behave as though they were positively charged, as in the case of the ions of the monovalent metals. Other atoms or radicles readily gain an electron and are therefore negatively charged as in the acid radicles. It is therefore possible that the chemical affinity between them is due to the attraction between these opposite charges. Anything which reduces the attraction between them would loosen the state of chemical combination. Water, with its high dielectric constant would certainly lessen this attraction (p. 60), which may account for the ready dissociation of many salts when dissolved in water.

If, then, a substance such as NaCl is partially dissociated into $\overset{+}{\text{Na}}$ and $\overset{-}{\text{Cl}}$ on being dissolved in water, an electric field between the electrodes (Fig. 100) would drive the positively charged sodium ions towards the cathode B, and the negatively charged chlorine ions towards the anode A. The movement of these ions through the solution, positive in one direction and negative in the other, is the current in the solution. When the positive ions reach the cathode they resume their ordinary neutral state and the charge is handed on. It is perhaps more correct to say that by receiving an electron or negative charge from the electrode they become neutral. Similarly, the negative ions on reaching the anode give up their negative charge and resume their neutral state.

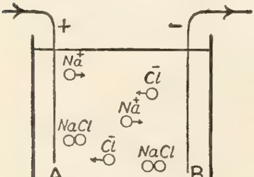


FIG. 100.—Diagram illustrating electrolysis.

It would therefore follow that the more completely a substance dissociates on being dissolved, the greater will be the conductivity of the solution, as there are more ions to carry the current. This is found to be the case, as there is evidence of a non-electrical character, of the dissociation, being derived from the raising of

the boiling-point or lowering of the freezing-point of the solution. This, however, cannot be discussed here. It may be noted, however, that when dissociation does not occur no current will pass and the solution is not an electrolyte.

Resistivity of electrolyte.—One of the simplest methods of measuring the resistivity of an electrolyte is to place it in a vertical tube and use flat electrodes which nearly fill the cross-section of the tube, as shown in Fig. 101. A battery, galvanometer and resistance box R complete the circuit, and the sensitiveness of the galvanometer is adjusted by means of a shunt or a control magnet, until a convenient deflection is obtained when there is no resistance in R. The electrodes are now pushed together through a measured distance l, and resistance introduced into R until the galvanometer deflection is restored to its original value. The current being then the same, the total resistance is the same as before, so that the resistance of length l of the electrolyte in the tube is equal to the resistance introduced by the box R. The area of cross-section of the tube may be found by pouring into it a known volume of water from a measuring vessel and measuring the length of the column of water added. Then, knowing the resistance of a known length and cross-section of electrolyte the resistivity can be calculated (p. 80). The temperature of the electrolyte at the time of measuring its resistance should be observed and recorded, as its resistivity depends upon the temperature. The resistance at the electrode is an unknown quantity, but this is present in both cases in which the current is observed, so that the actual resistance of the length l of electrolyte is found. Where greater accuracy is necessary, the tube may be made an arm of the Wheatstone's bridge, an alternating current from an induction coil being used instead of the direct current from a cell. In this case the ordinary galvanometer is useless for determining the balance and must be replaced by a telephone receiver.

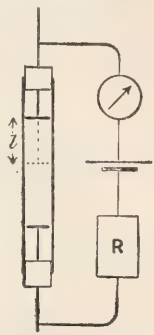


FIG. 101.—Resistivity of electrolyte.

Simple voltaic cell.—The earliest type of electrolytic cell used for the production of electric current is that due to Volta. It consists of plates of copper and zinc separated by an acid solution. Volta constructed a pile as shown in Fig. 102, by taking plates of copper and zinc separated by a piece of cloth moistened with acid, and building them up in series. Although the capacity of such a pile for producing current is small, the electromotive force is considerable, and

with 100 such elements in the pile, shocks may be obtained on touching the extreme electrodes.

The simple voltaic cell must now be studied in detail. If a zinc and a copper electrode dip into dilute sulphuric acid, the zinc dissolves and bubbles of hydrogen rise from it. If now the electrodes be connected externally by a wire, the hydrogen will be seen to bubble from the copper and a current flows, as may be proved by including a galvanometer in the external circuit (Fig. 103). Owing to the fact that the

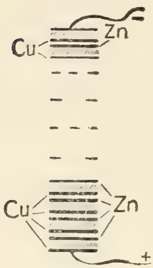


FIG. 102.—Voltaic pile.

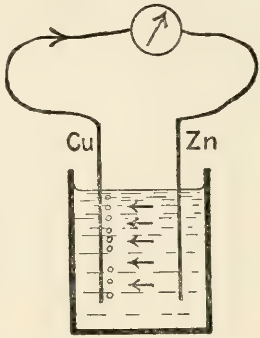


FIG. 103.—Simple voltaic cell.

hydrogen forms on the copper electrode, it follows that the current is from zinc to copper through the electrolyte, and from copper to zinc in the external circuit.

The current is carried by the hydrogen ions (+) and the SO_4 ions (−), while the SO_4 with the zinc forms zinc sulphate ($ZnSO_4$). When zinc forms zinc sulphate energy is liberated, which, in the ordinary process of dissolving zinc in sulphuric acid, is liberated in the form of heat. When the cell is producing current, part of the energy liberated by the solution of the zinc is used in maintaining the current and is eventually converted into heat in the current circuit.

Polarisation.—The simple cell is not very efficient for maintaining a current, the reason being that the hydrogen deposited upon the copper has two effects. Firstly, it reduces the effective conducting surface of the copper electrode and so increases the internal resistance of the cell. Secondly, it

produces an e.m.f. in opposition to the main e.m.f. This is called a **back electromotive force,** as it tends to send a current the reverse way round the circuit and so reduces the effective e.m.f. of the cell.

This reverse e.m.f. may be shown to exist by passing a current between platinum electrodes through a dilute solution of sulphuric acid, and connecting the electrodes to a galvanometer while the hydrogen is still upon the cathode. If *a* and *k* in Fig. 104 are the platinum plates, then on depressing the key K a current flows and the plates become covered, *a* with oxygen and *k* with hydrogen. On raising the key K, the battery is cut out and the galvanometer is connected to *a* and *k*. A current will be found to flow, for a time depending upon the amount of hydrogen deposited.

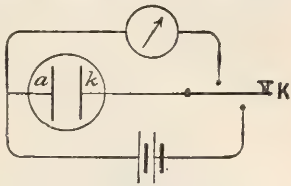


FIG. 104.—Experiment to illustrate polarisation.

When a back electromotive force exists on account of the deposition of hydrogen, the cell is said to be polarised. In designing a cell for the production of current, one of the most important objects is to avoid or remove the hydrogen which produces polarisation. In early times there were many such cells, but now that the public supply of current is nearly universal, only a few types of cell, useful for particular purposes, have survived.

Daniell's cell.—One of the most useful cells for the production of small currents in the laboratory is the Daniell's cell (Fig. 105). A porous pot separates two solutions, the outer one a strong solution of copper sulphate, and the inner one a dilute solution of sulphuric acid. In the former is an electrode of copper, whch may be the containing vessel itself, and in the latter is a zinc rod. Current flows in the external circuit from copper to zinc, the copper being usually called the positive electrode and the zinc the negative. Copper is therefore deposited upon the copper electrode, and zinc is dissolved with formation of zinc sulphate. There is consequently no polarisation, and the electromotive force of the cell is fairly constant, being from 1·07 to 1·1 volt. The two solutions come into contact within the walls of the porous pot, and hydrogen ions travel towards the copper,

forming with the SO_4, sulphuric acid. Thus the copper sulphate solution is gradually changed to sulphuric acid, and if the cell is used for long running, copper sulphate crystals are added to replace the lost copper sulphate.

Standard cells.—Certain cells, when constructed of pure chemicals, have very constant e.m.f. and are then used as standards. The most important of these standard cells is the **cadmium** or **Weston cell.** In Fig. 106 the arrangement of this cell may be seen. The positive electrode is a pool of mercury C, upon which rests a paste of mercurous sulphate

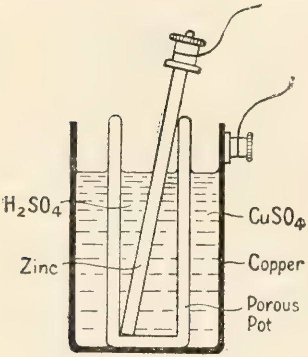


FIG. 105.—Daniell's cell

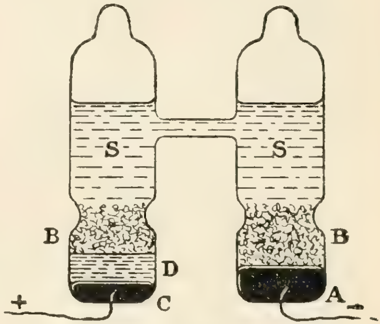


FIG. 106—Cadmium or Weston standard cell.

D, with cadmium sulphate crystals B. The negative electrode A consists of an amalgam of cadmium, 12 parts of mercury to 88 parts of cadmium. On the amalgam rests a paste of cadmium sulphate crystals, and the circuit is completed by a saturated solution of cadmium sulphate S. The whole is contained in a sealed glass vessel in the shape of the letter H. The electromotive force is 1·0183 volt at 20° C., and at any other temperature $t°$ C. the e.m.f. may be calculated from the relation—

$$E = 1{\cdot}0183 - 0{\cdot}0000406(t - 20) \text{ volt}$$

It is thus seen that the temperature coefficient is very small.

Care must be taken that the current flowing through the cell is always minute. In order to guard against the accidental production of current large enough to ruin the

cell, it is a frequent practice to connect a high resistance permanently in series with it.

Leclanché cell.—For purposes where a moderate current is required for a short time, with long intervals of rest, as for electric bells, telephones, etc., the cells of the **Leclanché** type are in very common use. The positive electrode is a carbon rod (gas coke) and the negative electrode is zinc, the electrolyte being a strong solution of ammonium chloride or sal-ammoniac (NH_4Cl). When the current passes, chlorine ions are liberated at the zinc, forming zinc chloride ($ZnCl_2$). At the carbon NH_4 is liberated, but this being an unstable substance forms ammonia (NH_3) and hydrogen,

$$2NH_4 = 2NH_3 + H_2$$

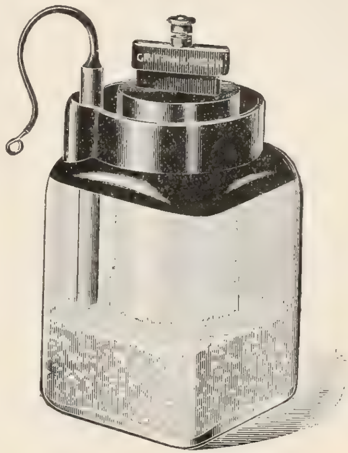


FIG. 107.—Leclanché cell.

The hydrogen deposited upon the carbon polarises the cell, but it is gradually removed by black oxide of manganese (MnO_2) which is packed into a porous pot and surrounds the carbon rod (Fig. 107). The MnO_2 parts with some of its oxygen, which forms water with the hydrogen, and so the cell gradually becomes depolarised on resting.

$$H_2 + 2MnO_2 = H_2O + Mn_2O_3$$

Dry cells.—Cells of the Leclanché type are usually employed in out-of-the-way places where they will not be disturbed. If, however, it is desired to carry them about, it is customary to pack the space round the manganese dioxide with a paste of sawdust and glycerine saturated with a solution of ammonium chloride. This will retain sufficient moisture for the effective working of the cell. As there is no free liquid to spill, this is generally called a **dry cell.**

Amalgamation of zinc.—Whenever a zinc rod is used for the negative electrode of a cell it is customary to rub it over with

mercury, thus forming an outer layer of zinc amalgam in contact with the electrolyte. The reason is that commercial zinc contains impurities, one of which is iron, and pure zinc is too expensive to use. A piece of impurity such as iron in the surface of the zinc, forms with the electrolyte and the zinc a local cell, and the current flowing in it will rapidly dissolve the zinc. This is very wasteful as the zinc becomes pitted and rapidly wastes away, even when the cell is not producing external current. By amalgamating the zinc, a uniform surface is presented to the solution, any impurities being covered over, and the zinc will not then waste away. The action of the cell is in no way impeded, because the zinc in the amalgam is in contact with the electrolyte and dissolves in the ordinary way when the cell is producing current.

Secondary cells or accumulators.—All the cells described up to now have one feature in common, the materials are put together and the cell is immediately capable of producing a current. Such are usually called **primary cells.** Those cells which are not capable of producing a current until they have been **charged** by passing a current through them, are called **secondary cells** or **accumulators.** By far the most common type of secondary cell consists of two lead plates immersed in a dilute solution of sulphuric acid. On passing a current through such a cell, hydrogen bubbles up from the cathode, and the anode is oxidised to form lead peroxide (PbO_2), which forms the positive plate of the cell.

On joining the electrodes by an external wire, a current passes externally from positive plate to negative, the positive being now the cathode. The hydrogen liberated reduces the PbO_2, thus—

$$PbO_2 + H_2 + H_2SO_4 = PbSO_4 + 2H_2O$$

At the same time SO_4 liberated at the anode forms with the lead plate $PbSO_4$. Thus when the current passes, both plates become coated with lead sulphate ($PbSO_4$) and on reaching the same state the current ceases.

Forming the plates.—Starting with lead plates and performing a single charging, it is found that the cell has very little storage capacity. After a very short time a layer of PbO_2 is formed which protects the lead beneath, and oxygen bubbles up from the anode. It is useless to attempt to carry the charging beyond this point. It was found, however, by Planté in 1859 that if on letting the cell discharge, the current be continued in the

discharging direction by applying some external electromotive force, the $PbSO_4$ on the positive is reduced to lead—

$$PbSO_4+H_2=Pb+H_2SO_4$$

and the $PbSO_4$ on the negative is oxidised to PbO_2—

$$PbSO_4+SO_4+2H_2O=PbO_2+2H_2SO_4$$

The cell is now reversed, and may be discharged, and then recharged in the original direction. Every time that this reversal of charging is carried out, the lead formed is in a spongy layer which eats deeper and deeper into the lead plate. The electrolyte permeates the spongy lead, so that at each charging more lead takes part in the process and the storage capacity of the cell is increased. This process of increasing the storage capacity of the cell is called **forming** the plates.

Many devices are employed for obtaining a large surface of lead to begin with. Some plates are built up of strips, welded into a lead frame, and others are made of plates with holes into which corrugated strips of lead are stamped.

Paste plates.—The process of forming the plates takes some time and is expensive. To shorten the process of forming, Faure suggested that a mixture of several oxides of lead with sulphuric acid, made into a paste, should be stamped into a framework or grid of lead. These grids are of many forms, but the good ones are so designed that the paste keys into the plate. The plates so constructed are charged and recharged as for the formed plate, but the process is much shorter. Paste plates are not so strong as formed plates, but are cheaper. Some manufacturers use formed plates as positives and paste plates as negatives.

Use of accumulators.—In Fig. 108 is illustrated a large storage cell consisting of five positive plates and six negative plates. The plates alternate, and all the positives are connected together by a stout lead bar to which they are welded, and the negatives are connected together in a similar manner. The whole set of plates is supported upon insulators in a strong glass box which contains the dilute sulphuric acid. This should have a density of 1·21 when the cell is fully charged. The density of the electrolyte is a good guide to the condition of the cell, for during the charging, sulphuric acid passes into solution and the density rises. During discharge, sulphuric acid passes out of the solution to form lead sulphate, and the density falls. It should never be allowed to fall below 1·15 as there is a tendency then for the lead sulphate to pass from the active form which it has in an efficient cell to that of a hard white solid, which cannot be reduced or oxidised by the passage of a current. If this

should happen the cell is said to be **sulphated** and is **ruined.**

From the shape of the cell it will be seen that the electrolyte is in the form of a comparatively thin layer of considerable area between the plates, and as the solution is a fairly good conductor, the internal resistance of the secondary cell is very small. It is on this account that the cell is so much used when large currents are required.

Consider a Daniell's cell whose e.m.f. may be 1·1 volt and internal resistance, say, 1 ohm. The greatest current such a cell can produce occurs when the cell is **short-circuited,** that is, when

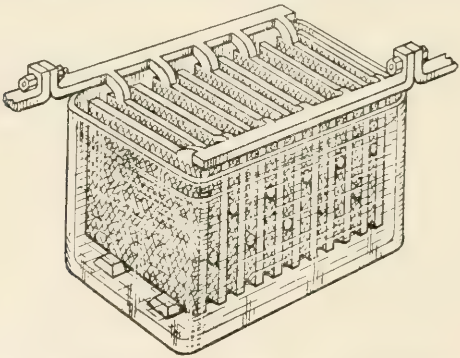


FIG. 108.—Secondary or storage cell, or accumulator.

the cell terminals are joined by a conductor of very low resistance, so that the external resistance may be taken as zero.

Then, $$\text{current} = \frac{1\cdot 1}{1} = 1\cdot 1 \text{ ampere}$$

Now consider a secondary cell of e.m.f. 2·1 volts, whose internal resistance may be as low as a hundredth of an ohm. When the cell is short-circuited—

$$\text{current} = \frac{2\cdot 1}{0\cdot 01} = 210 \text{ amperes}$$

Of course such a current should not be allowed to pass—it would injure the cell unless it were one of very large area of plate but the illustration shows that almost any current desired can be produced. Batteries of accumulators are often used in conjunction with a dynamo for lighting purposes. A battery of 50 cells provides an e.m.f. of 105 volts. When the dynamo is running it provides the current for lighting and also enough for the charging

of the accumulators. When the dynamo is not running, the lighting current is drawn from the accumulators.

The secondary cell has an electromotive force which is very nearly constant at 2·1 volts during the greater part of the time of discharge. During charging, the e.m.f. is higher and may reach 2·5 volts at the completion of charge, but it rapidly falls to 2·1 volts when charging ceases. When, on using current, the e.m.f. falls to 2·0 volts the cell is nearly empty. It should then be charged at once, as it is highly injurious to the lead accumulator to stand uncharged.

Edison accumulator.—A secondary cell of a very different type from the above has been devised by Edison. The positive electrode consists of nickel tubes carried on a nickel framework and packed with nickel hydrate. The negative consists of a nickel-steel sheet having pockets containing finely-divided iron oxide, and the electrolyte is a solution of potassium hydrate. It is claimed that the Edison accumulator is much more robust than the lead accumulator, and can stand uncharged for considerable periods without deteriorating. Also it is lighter than the lead accumulator, but it has the disadvantage of a low e.m.f., 1·2 volt. Thus for a 100-volt supply it would be necessary to have 84 Edison cells, whereas 50 lead cells give 105 volts.

Aluminium rectifiers.—The metal aluminium has peculiar properties when used as an electrode. It will only allow current to pass in one direction, which is, from the electrolyte to the aluminium. Thus aluminium will act as a cathode but not as an anode. If, therefore, a cell consisting of a lead and an aluminium electrode immersed in dilute sulphuric acid be placed in a circuit in which an alternating current (p. 157) is flowing, the current in one direction is suppressed while that in the opposite direction is allowed to flow. Such a cell is called a **rectifier**. This phenomenon is of considerable use in the charging of small accumulators where the public supply of current is alternating. If the supply is at 100 volts or more, it is advantageous to use a transformer (p. 159) to reduce the voltage to 10 or 20 volts, according to the number of cells to be charged in series. This transformer consists of many turns of fine wire C wound round a bundle of iron wires (Fig. 109). The few turns of thick wire D are then connected to the cells S to be charged and the rectifier R as shown. The current will then flow in the proper charging

direction through the cells, the reverse current being suppressed by the rectifier.

If thirty or forty cells are to be charged in series, the transformer may be dispensed with. A more efficient but more troublesome arrangement for charging is shown in Fig. 110 (*a*). Here four rectifying cells are used, and either direction of the alternating e.m.f. of the supply, applied at A and B, sends a current in the proper charging direction through the secondary cells S. Thus for one alternation the current flows in the path ARSQB, and for the other alternation the path is BTSPA. The current cannot pass directly from A to B without going through the accumulators, for it is always suppressed by one of the rectifying cells.

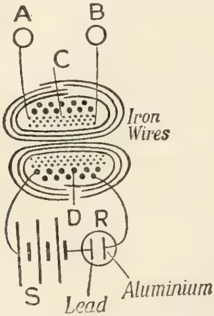


FIG. 109.—Transformer for charging accumulators.

If a connection can be made to the middle of the low voltage transformer coil, the connections may be made as in Fig. 110 (*b*), where only two rectifying cells are required. It will be seen that the current is always in the proper charging direction for the accumulators S, the reverse current being suppressed.

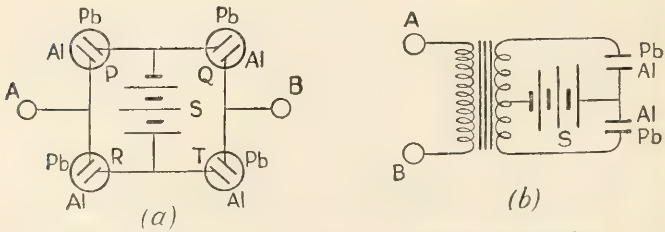


FIG. 110.—Charging accumulators by aluminium rectifiers.

Various substances may be used as electrolyte for the rectifier, but a strong solution of sodium bicarbonate is for some reasons the best. It is very efficient and the lead may be replaced by iron if desired. The chief objection to sodium bicarbonate is that the electrolyte becomes choked up with a gelatinous precipitate of aluminium oxide. However, sodium bicarbonate is fairly cheap and the solution may be renewed frequently. Other substances that have given good results are borax and ammonium phosphate.

EXERCISES ON CHAPTER VIII

1. Describe what you would observe on passing an electric current between platinum electrodes, through solutions of sulphuric acid, copper sulphate, and silver nitrate.

2. Give an account of the passage of current through a solution, giving the terms usually employed.

3. State Faraday's laws of electrolysis.

Explain how they are made use of in the measurement of electric current.

4. What is meant by "the electrochemical equivalent" of a substance?

A current passing through acidulated water liberates hydrogen, which, when reduced to standard conditions, has a volume of 2·5 litres. If the current passed for $1\frac{1}{2}$ hours and the density of hydrogen is 0·0896 grm. per litre, what is the strength of the current?

5. Describe carefully the use of the copper voltameter for the measurement of current.

6. A tangent galvanometer having a coil of 6 turns and radius 18 cm. is placed in series with a copper voltameter. If the current flows for half an hour and 0·42 grm. of copper is deposited, the deflection all the time being 30°, what is the value of the earth's magnetic field?

7. An ammeter reads 0·92 ampere, and the weight of copper deposited in a copper voltameter in series with it is 0·801 grm., the current flowing for 45 minutes. What is the correction to be applied to the ammeter?

8. Give a short account of the dissociation theory of electrolysis.

9. Describe some method of measuring the resistivity of an electrolyte.

10. Describe the simple Voltaic cell, and explain why it is not efficient as a producer of current.

11. Describe some form of cell having fairly constant electromotive force and give the reason of the constancy.

12. Explain the processes occurring in a Leclanché cell. What are the uses of such a cell?

13. Give the reason for the custom of amalgamating the zinc electrodes used in cells.

14. What is a secondary cell or accumulator?

Describe some form of this type of cell.

15. Describe how an aluminium rectifier may be used to charge accumulators from an alternating current source of supply.

16. A current passes through two cells in series, one containing copper sulphate solution and the other a sodium salt. If the current passes until 12·5 grms. of copper is deposited, how much sodium is liberated?

CHAPTER IX

ELECTRODYNAMICS

Reaction between current and magnetic pole. — In Chapter III the magnetic field which accompanies an electric current has been studied. A magnetic field involves force on a magnetic pole, in fact the magnetic field is measured by the force on a unit N pole placed in the field. If, then, the magnetic field is due to an electric current, it follows that the force on the pole due to the current has a counterpart in a force on the current due to the pole. According to the laws of dynamics, whenever there is force between two bodies, the force on one is equal and opposite to the force on the other.

Let us apply this law to the case of a circle of wire carrying current, with a magnetic N pole situated at its centre. If i is the current, m the strength of pole, and r the radius of the circle in centimetres, then the strength of magnetic field at m due to the current is $\frac{2\pi i}{r}$ (p. 25), and the force on m is $\frac{2\pi im}{r}$ dynes (Fig. 111). It follows from the laws of dynamics that there is a force of $\frac{2\pi im}{r}$ dynes acting upon the coil in the opposite direction to that acting upon m. This force on the coil acts all over it, and the force on each centimetre of the coil is therefore $\frac{2\pi im}{r} \times \frac{1}{2\pi r} = \frac{im}{r^2}$, because $2\pi r$ cm. is the circumference of the circle.

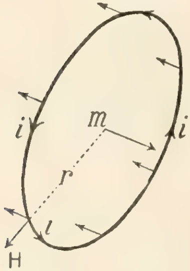


FIG. 111.—Force on current.

Now m/r^2 is the strength of magnetic field at distance r cm. from the magnetic pole m, so that if $\frac{m}{r^2} = H$

Force per centimetre of conductor $= i$H dynes

Force on current in magnetic field.— The above result represents an important law in electrodynamics, namely that **whenever a current i flows at right angles to a magnetic field of strength H, there is a force of iH dynes acting on each unit of length of the current, at right angles to both field and current.**

Strictly speaking, the force on the current depends upon the magnetic induction B and not the field H (p. 21), but since in almost every case the currents we shall deal with are carried by conductors situated in air, whose magnetic permeability is unity, B and H are numerically equal (p. 227) and we shall keep to the practice of using H in our calculations.

The direction of the force upon the current depends upon the direction of both current and magnetic field and may be seen from Fig. 111. Another way of remembering the rule for the direction of the force is illustrated in Fig. 112. H, i, and the force are represented by three edges of a cube. Then if we look along the magnetic field, it would require a **counter-clockwise** rotation to change the direction of the current to that of the force upon the current.

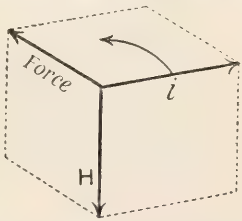


FIG. 112.—Rule for direction of force on current.

If the current and the magnetic field are not at right angles to each other but include an angle θ between them, the law of force is more general than the above ; it is—

Force per cm. of current $= i$H $\sin\theta$ dynes

It will be seen that when $\theta = 90°$ this reduces to iH dynes as before, also when $\theta = 0$, $\sin\theta = 0$, and there is no force on the current when its direction is the same as that of the magnetic field.

Couple on coil in magnetic field.—Consider a vertical rectangular coil *abcd* (Fig. 113 (i)) situated in a horizontal magnetic field H. The force on the top, *ad*, will be upwards,

and that on the bottom, *bc*, will be downwards, so that these two, being in the same straight line, will cancel and will not give rise to a couple. The forces on the vertical sides may best be understood by taking a plan (Fig. 113 (ii)). The forces are shown by arrows at *a* and *d*, and it will be seen that the two give rise to a couple tending to twist the coil. Each force has the value $i\mathrm{H}(ab)$, being $i\mathrm{H}$ dynes per centimetre length of side. The perpendicular distance between the forces is $(ed)=(ad)\sin\theta$—

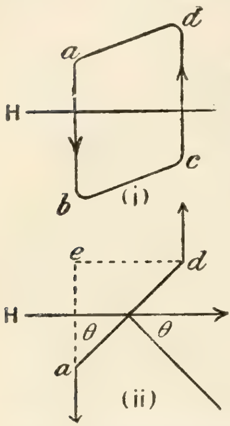


FIG. 113.—Coil in magnetic field.

$$\therefore \text{Couple}=i\mathrm{H}(ab)(ed)=i\mathrm{H}(ab)(ad)\sin\theta$$
$$=i\mathrm{HA}\sin\theta$$

where $\mathrm{A}=ab\times ad$, the area of the coil.

On comparing this with the couple on a magnet in a magnetic field, $\mathrm{HM}\sin\theta$ (p. 14), it will be seen that the coil of area A carrying current i experiences exactly the same couple in a magnetic field as though it were a magnet of magnetic moment $i\mathrm{A}$, with its magnetic axis perpendicular to the plane of the coil.

Suspended coil galvanometer.—The coil suspended in a magnetic field affords the principle on which most modern galvanometers are made. If *ad* (Fig. 114) represents the coil, carrying current i, and NS the poles of a powerful permanent magnet which produces a magnetic field of strength H at *a* and *d*, then the couple acting on the coil is $i\mathrm{HA}$, for $\theta=90°$, the normal to the plane of the coil being perpendicular to the magnetic field. If the coil be suspended by a fine wire (Fig. 115), then if the coil rotates, the suspension wire twists, and the coil comes to rest when the couple due to the twist in the suspension is equal to that due to the current in the coil. The couple due to the twist in the suspension is proportional to the rotation θ, of its lower end—

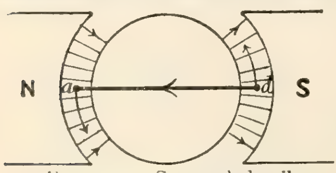


FIG. 114.—Suspended coil.

Thus, couple $=c\theta$, where c is the couple for unit twist

$$\therefore\ c\theta = i\mathrm{HA}$$

$$\therefore\ \theta = \frac{\mathrm{HA}}{c}\, i$$

That is, the deflection is proportional to the current, provided that the magnetic field is still of the same value and still parallel to the plane of the coil when the coil has changed its position. This is ensured by hollowing the pole pieces to a cylindrical shape and placing between them a soft iron cylinder E (Fig. 115). This makes the magnetic field radial in form and renders the scale of the galvanometer linear, that is, the deflection is proportional to the current.

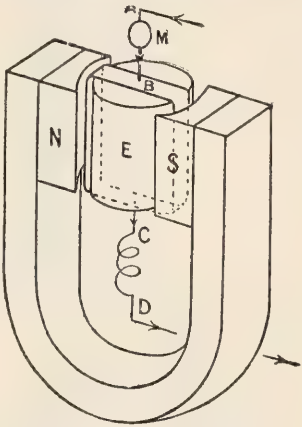


FIG. 115.—Suspended coil galvanometer.

The suspension is usually a fine phosphor-bronze strip or ribbon, and carries the current to the suspended coil. A second, loose strip at the bottom takes the current away. Sometimes, however, the two strips are side by side and both support the coil. This is called a bifilar suspension.

It should be remembered that the area A in the above expression for the deflection is the effective area of the coil. If the coil has many turns, as is usually the case, A is the effective area of all the turns, or the number of turns multiplied by the average area of one turn.

A very good form of suspended coil galvanometer is illustrated in Fig. 116. This pattern follows closely the diagrammatic form in Fig. 115. In many other patterns, different designs are employed. In the case of moving coil ammeters and voltmeters (p. 89), a spring control is used, in order to make the instrument less sensitive and less liable to damage.

Law of work on carrying pole round a current.—It is possible from the equivalence of current circuits and magnets, established by A. M. Ampère, to deduce a law which is of considerable use. It is beyond the scope of this book to

give the proof here. The law is—that **if a unit magnetic pole be carried once round a current of strength i, the work done is $4\pi i$ ergs.**

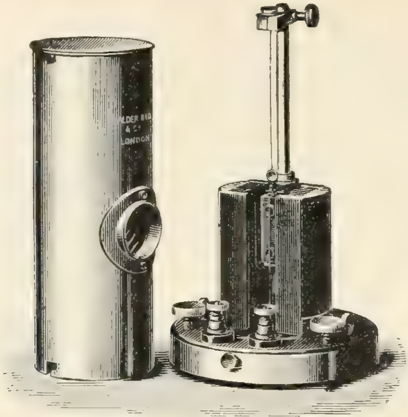


FIG. 116.—Suspended coil reflecting galvanometer.

Let us apply this law to finding the strength of magnetic field near a long straight wire carrying a current i. From our knowledge of the form of the magnetic field near a straight current (p. 24) it is clear that the field has the same strength, H, at equal distances from the wire. Let a unit pole at P (Fig. 117) be situated at distance r from the current. The force on the pole is H and is always tangential to the circle. If the pole is now carried once round the circle whose centre is on the wire—

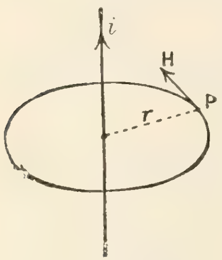


FIG. 117.—Magnetic field due to straight current.

$$\text{Work done} = \text{force} \times \text{distance}$$
$$= \text{H} \times 2\pi r$$

But by the above law, the work done is $4\pi i$,

$$\therefore \ \text{H}.2\pi r = 4\pi i$$
$$\text{H} = \frac{2i}{r}$$

Thus the strength of field varies inversely as the distance from the current.

Force between currents.—We are now in a position to determine the force which one current exerts on another, in certain simple cases. Consider two straight parallel wires r cm. apart carrying currents respectively i_1 and i_2 (Fig. 118). Then the magnetic field H at the second wire due to the first is $H = \frac{2i_1}{r}$.

Therefore, the force on each centimetre of the second due to this field is $i_2H = \frac{2i_1i_2}{r}$ dynes, and the rule for direction (p. 133) shows that the force is directed along F. Similar reasoning will show that the same force per cm. length of

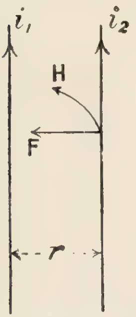


FIG. 118.—Force between parallel currents.

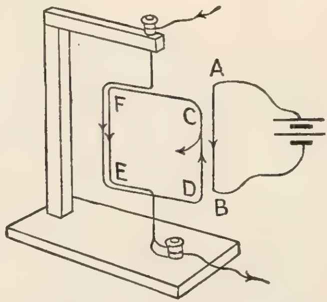


FIG. 119.—Force between currents.

wire drives the first towards the second. We can therefore say that—

currents in the same direction attract each other.

If the currents had been in opposite directions the force would be the same in magnitude, but the conductors would be driven apart. Thus—

currents in opposite directions repel each other.

A simple experiment will prove these facts. The coil CDEF (Fig. 119) is suspended by a fine wire which carries the current, but will allow the coil to rotate. A wire AB carrying a fairly strong current is brought near the side CD of the coil. It will then be found that repulsion exists when the currents in AB and

CD are in opposite directions, but attraction when the currents are in the same direction. The same apparatus may be used to show the force on a current in a magnetic field (p. 133), by bringing one pole of a bar magnet near either CD or EF.

Kelvin current balance.—The force between currents has been used in several current-measuring instruments, the best known of which is the Kelvin current balance, which is shown diagrammatically in Fig. 120. The current to be measured passes through four fixed coils A, B, C and D in series. It also passes through two coils E and F in series with the other four. E and F are fixed to the arms of a balance which also carries a straight scale with a sliding weight W. The coils are so connected together that E is forced downwards and F upwards when the current passes. Equilibrium is restored by sliding the weight W to the right, in this way producing a restoring couple which balances the deflecting couple due to the current. The force acting upon E or F due to each fixed coil is proportional to the product of the currents in the two. But as these currents are the same, each force, and therefore the whole couple, is proportional to the square of the current flowing. The scale along which W slides is calibrated to read amperes, deci-, or centi-amperes, according to the weight of W.

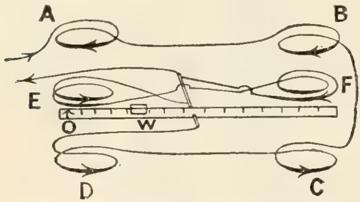


FIG. 120.—Kelvin current balance.

If the current is reversed, the couple is still in its old direction, for the current in every coil is reversed. The instrument will therefore read with alternating as well as with direct currents.

Kelvin watt-balance.—It is possible to modify the current balance so that the power used in a circuit can be measured in one reading. If the six coils of the balance are connected as in Fig. 121, the current I, in the lamp L, the power in which is to be measured, flows through the four fixed coils A, B, C and D. E and F are connected to M and N, so that the current in them is proportional to the potential difference, E, between the ends of the lamp. Thus each

force on E and F is proportional to the current in L and to the p.d. across L.

$$\therefore \text{ Couple} \propto I \times E$$
$$\propto \text{ watts used in lamp.}$$

The arm along which the weight slides can therefore be calibrated to read directly in watts. It is necessary in

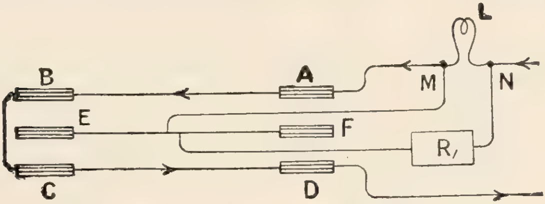


FIG. 121.—Kelvin watt-balance.

determining the scale, to measure the watts for some particular scale reading, by using an ammeter and a voltmeter, and finding the watts in the ordinary way (p. 96). From this reading the value of the watts for any other position of the sliding weight becomes known.

It should be noted that the resistance of the series coils A, B, C and D must be small, for the same reason that the resistance of an ammeter must be small (p. 91). Also the shunt coils F and E must have high resistance, as they are placed in a similar position to a voltmeter (p. 91). A resistance R_1 may be included in this circuit in order to increase its total resistance.

There are many other forms of watt-meter, but their principle is the same as in the case of the Kelvin instrument.

Electromotive force due to cutting across magnetic field.—One of the most important discoveries of Michael Faraday was that when a conductor cuts across a magnetic field, an electromotive force is produced in the conductor. This fact, however, may be deduced directly from our knowledge of the force acting upon a current in a magnetic field.

Consider a conductor AB, of length l cm. in which a current i is flowing, to be situated at right angles to a magnetic field H (Fig. 122). It has been seen that a force of iH dynes per cm. acts on the conductor; the whole force is then iHl dynes, and is at right angles to both AB and H. If this

force moves the conductor with velocity v cm. per second in the direction of the force, either by sliding contacts or otherwise, the work done in one second is force$\times$distance$=i\mathrm{H}lv$ ergs. The source of supply of energy is the battery e, which provides energy to the circuit at the rate of ei ergs per second (p. 74). This energy is used in moving the conductor and in heating the circuit, the latter process requiring i^2r ergs per second

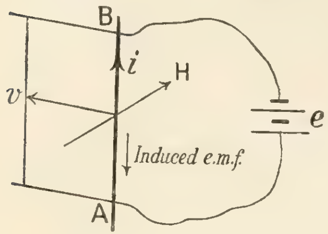


FIG. 122.—Induced e.m.f.

$$\therefore\ i^2r+i\mathrm{H}lv=ei$$
$$ir+\mathrm{H}lv=e$$
$$i=\frac{e-\mathrm{H}lv}{r}$$

This means that the effective e.m.f. in the circuit is reduced by amount $\mathrm{H}lv$ on account of the motion of the conductor. This e.m.f. $\mathrm{H}lv$ is that produced by the cutting of the conductor across the magnetic field. Also lv is the area swept out by the conductor in one second, and H is the number of magnetic lines of force per square centimetre; hence—**the electromotive force is the rate of cutting magnetic lines of force,** or the number of magnetic lines of force cut per second.

The direction of the induced e.m.f. in the conductor cutting across a magnetic field may be deduced from Fig. 122. For, from the above equation it is evidently in opposition to the e.m.f. of the battery and is therefore directed downwards, that is from B to A in the figure. The rule is therefore, look along the magnetic lines of force, and a rotation of the direction of the velocity in an **anti-clockwise** sense will bring it to the direction of the induced e.m.f. By comparing with the rule for force on a conductor (p. 133) it will be seen that there is really only one rule to remember—

Look along the lines of force, then an **anti-clockwise** rotation brings direction of **current** into direction of **force** on conductor, or direction of **motion** into direction of **induced e.m.f.,** that is, will rotate the **cause** into the direction of the **effect.**

Induced e.m.f. in complete circuit.—It must be clearly

understood that the induced e.m.f. is due to the cutting of conductor and magnetic lines of force, and it does not matter whether the field is fixed and the conductor moves, or the conductor is fixed and the magnetic field moves ; it is the relative motion of the two that determines the e.m.f. If ABC (Fig. 123) is a conducting circuit with its plane at right angles to a magnetic field of strength H, then the field may be cut by the conductor by a motion in its own plane, in which case the e.m.f.'s in the various parts of the circuit would cancel each other and there would be no resultant e.m.f. acting round the circuit. At the same time, the total number of lines threaded through the circuit is unchanged.

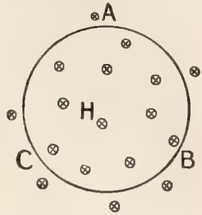


FIG. 123.—E.m.f. in complete circuit.

If, however, the number of lines threaded through the circuit changes, this may take place either by a change in size of the circuit or by a change in strength of the field, by the lines moving inwards or outwards. In the former case they become concentrated within the circuit and in the latter they are more separated. In any case the total number of lines of force cut per second is the change per second in the number of lines threaded through the circuit, and is the electromotive force directed round the circuit. By applying the rule on p. 140 it may be found that if we **look along the lines of force,** then if the number of lines threaded through the circuit is **increasing,** the induced e.m.f. is **anti-clockwise.** If the number of lines is **decreasing**, the induced e.m.f. is **clockwise**.

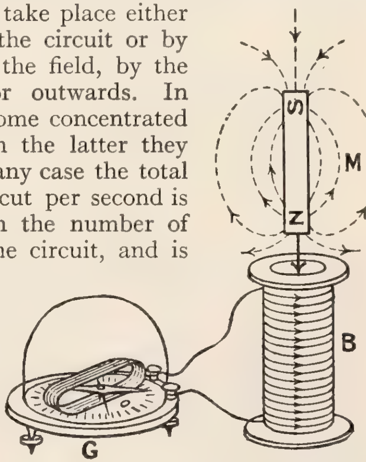


FIG. 124.—Experiment representing the production of current by electro-magnetic induction.

This fact may be shown by an experiment illustrated in Fig. 124. A solenoid B is connected in series with a galvanometer

G. On advancing the bar magnet M towards the coil, the number of magnetic lines of force threaded through the coil is increased. This causes an electromotive force which acts round the coil and produces current, because the circuit is complete. The galvanometer needle will be deflected throughout the whole time that the magnet is advancing. If now the magnet be withdrawn, there will be a current in the opposite direction to that on advancing, because the number of lines of force threaded through the coil is now decreasing. If the S pole of the magnet is directed towards the coil the currents and deflections are in the reverse direction to those obtained with the N pole directed towards the coil.

Telephone receiver.—One of the most important and ingenious applications of the production of e.m.f.'s by varying magnetic field was made by A. Graham Bell for the

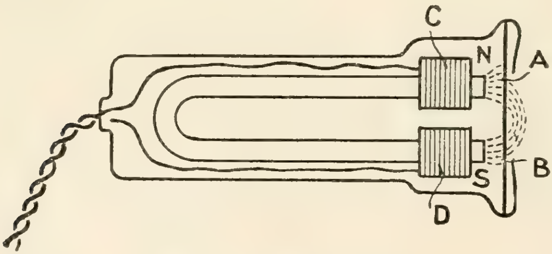


FIG. 125.—Bell telephone.

reproduction of sounds. This goes under the name of the telephone. Coils C and D (Fig. 125) are wound upon the poles NS of a magnet, and near the poles is situated a thin iron diaphragm AB. As the air waves, consisting of compressions and rarefactions, originated by the sounding body fall upon the diaphragm they drive it in and out rapidly. Magnetic poles are produced upon AB by the presence of the magnet NS, and as the diaphragm vibrates these poles are moving nearer to and further from the coils C D. Hence, if C and D form part of a complete conducting circuit, intermittent currents in tune with the vibrations of the sounding body are produced in them.

If the circuit is completed through long wires connected to the coils of an exactly similar instrument at a distance, the intermittent currents in the coils cause an intermittent

increase in the pull of the poles N and S upon the diaphragm AB of the distant instrument. The movement of the diaphragm sets up air waves which are the counterpart of those originally sent out by the sounding body. The instrument at the station at which the sounds originate is called a **transmitter,** and that at the receiving station is called a **telephone receiver.** The Bell receiver, although modified in design, is still the one generally used, but it is no longer used as a transmitter.

The magnet in the receiver must be a permanent magnet, for the pull on the diaphragm is proportional to the product of the strength of poles at N and A, that is to the square of the strength of poles at N, for that at A depends upon that at N. If, then, the pole at N has strength M, the pull on the diaphragm is proportional to M^2. Now the intensity of sound produced depends upon the **variation in pull,** and if the current in C is feeble, as is always the case, it will cause an additional pole strength in M equal, say, to m. Then when current flows, the pull on the diaphragm is proportional to $(M+m)^2=M^2+2Mm+m^2$. m^2 is a very small quantity, so that we may say that the variation in pull due to the telephone current is proportional to $2Mm$. That is, the stronger the original pole M, the greater will be the sounds produced. For this reason powerful permanent magnets are always used.

Carbon microphone.—A much more sensitive transmitter than the Bell instrument is the **carbon microphone** due to D. E. Hughes. This depends upon the fact that any mechanical disturbance makes great alteration in the resistance of the contact between pieces of carbon. In the telephone microphone, the current from a few dry cells passes from a block of carbon B (Fig. 126) to a carbon diaphragm D, through a pile of carbon granules G. As

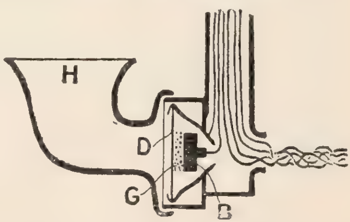


FIG. 126.—Carbon microphone.

the diaphragm vibrates, the resistance changes and imposes variations on the current corresponding to the vibrations of the sounding body. These cause vibrations in the

diaphragm of the distant Bell telephone receiver as described above.

Mutual inductance.—Since an electric current is accompanied by a magnetic field, it follows that as the current in any circuit grows, its magnetic field gets more intense. If there is a second circuit in the neighbourhood of that in which the current is growing, the magnetic field in growing cuts the second circuit and produces an electromotive force in it. If the current in the circuit A (Fig. 127) grows from zero to its final steady value on depressing the key, its accompanying magnetic field, part of which is threaded through the circuit B, also grows, and the number of magnetic lines of force threaded through B increases, and there will be an electromotive force in it which will produce current if the

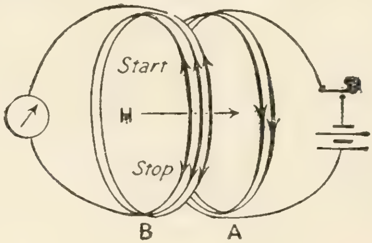


FIG. 127.—Circuits having mutual inductance.

circuit is complete. By applying the law on p. 141, it will be found that the induced current is in the **opposite direction** to the current in A, while the current in A is growing. If now the key be opened, the current in A dies away, and it will be found that while this is happening, the induced current in B is in the **same direction** as the current in A. Of course, any current in B has a magnetic field, and it will be noticed that the induced current in B is always in such a direction that it **tends to prevent any change in the total magnetic field.**

These effects may be illustrated by an experiment similar to the last (p. 141). If the coil A (Fig. 128), which has a current in it, be advanced towards the coil B, it will produce an induced current in it, indicated by a throw of the galvanometer needle or coil. This is to be expected because the solenoid A is equivalent to a magnet when current is flowing in it (p. 23).

The experiment may be continued by placing A inside B and keeping it there, when on starting the current in A, there will be a galvanometer throw exactly like that produced previously by pushing A into B while the current was flowing in A. Similarly, on stopping the current in A the effect is the same as withdrawing the coil.

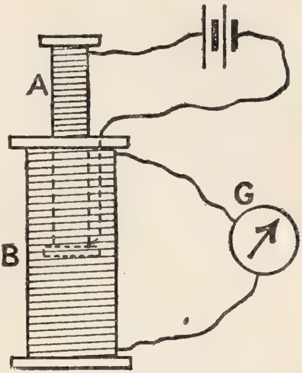


FIG. 128.—Experiment to illustrate the inductive effect of one circuit upon another

The induced e.m.f.'s and currents may be increased greatly by placing in A a soft iron rod or a bundle of iron wires, because this has the effect of increasing enormously the number of magnetic lines of force threaded with the two coils.

Coefficient of mutual induction.—The two coils A and B in Figs. 127 and 128 are usually called the **primary** and **secondary** coils respectively. A particular name is given to the electromotive force in the secondary coil when there is unit rate of change of current in the primary coil; it is the **coefficient of mutual induction** or the **mutual inductance** of the two coils. Thus if there is 1 unit of e.m f. in the secondary due to a change of 1 unit of current per second in the primary, the mutual inductance is unity. Thus—

$$\text{Mutual inductance} = \frac{\text{e.m.f. in secondary}}{\text{rate of change of current in primary}}$$

If the e.m.f. and current are measured in absolute C.G.S. units the mutual inductance is in absolute C.G.S. units. But if the e.m.f. is measured in volts and the current in amperes, the corresponding unit of mutual inductance is called the **henry.** Thus—

$$1 \text{ henry} = \frac{1 \text{ volt}}{1 \text{ ampere per second}}$$

$$= \frac{10^8 \text{ absolute units of e.m.f.}}{10^{-1} \text{ absolute units of current per second}}$$

$$= 10^9 \text{ absolute units of inductance}$$

Of course the actual growth of the current is usually

complete in much less time than a second, but although this is so, its rate of growth at any instant may be measured in amperes per second.

Induction coil.—The principle of mutual inductance has many applications, but one of the earliest is the **induction coil,** by means of which very high voltages are produced, for a very short interval of time, from a few cells of comparatively low voltage.

The induction coil has two circuits, the primary consisting of comparatively few turns A (Fig. 129) and the secondary B of a great number of turns of fine wire. Since the primary coil has to carry large currents it consists of thick wire, while the secondary must be made of fine wire in order to allow of a sufficient number

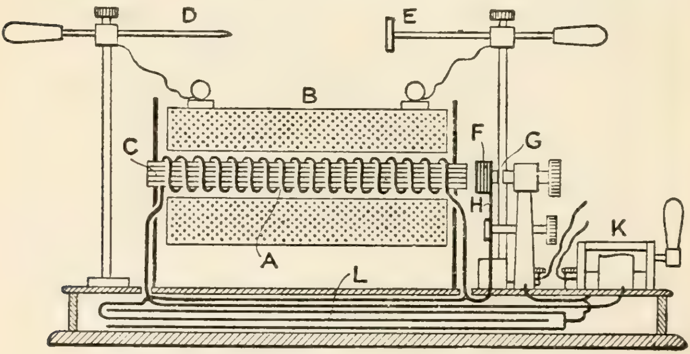


FIG. 129.—Induction coil showing the electric circuits.

of turns in a reasonable space. In the axis of both coils is an iron core C consisting of a bundle of soft iron wires. On starting or stopping the current in the primary coil, the magnetic field cuts the secondary, and since all the turns of the secondary are in series, a very high electromotive force is produced. Owing to the fact that the resistance of the primary circuit is much greater when the circuit is broken than when it is closed, the current in it dies away much more quickly than it grows. Thus the magnetic field is removed from the secondary much more rapidly than it is introduced, and the resulting e.m.f. in the secondary is much higher when the primary current is stopped than when it is established. When the primary current is interrupted, it is quite possible that the e.m.f. is sufficiently high to cause the secondary current to jump the air gap between the terminals E and F, although

this would not be the case on the establishment of the primary current.

In order to produce repeated interruption of the primary current automatically, a piece of soft iron F is carried on a spring H. When the primary current, which flows through the contact G, magnetises the soft iron core, F is pulled towards the core and the contact at G is broken and the spark takes place between E and D. But the core being now demagnetised, the spring H recoils and contact is again made at G. Thus the make and break is repeated automatically at a speed depending upon the stiffness of the spring H and the mass of the head F. The same device is used in electric bells and in the buzzers frequently used in telephony.

Another device which increases the efficiency of the induction coil is the introduction of a condenser L in parallel with spark gap G. The condenser, with the primary coil forms a circuit in which the current does not merely die away, but oscillates first, the oscillations dying away eventually (p. 176). Thus, on breaking the contact at G, the primary current is actually reversed, owing to the presence of the condenser. Thus the magnetic field is not only removed from the secondary but is put in in the reverse direction. The effect of the condenser is complicated, but the above will serve as an approximate explanation of its function.

Magneto.—In the older forms of petrol motor, the explosive gases, after being compressed in the cylinder, were fired by a spark from the secondary of an induction coil, the primary current being produced by a few accumulators. The objection to this system is the accumulator, which requires frequent charging and is always liable to give out. To get over this objection the **magneto** has come into use, which is really an induction coil whose primary current is produced by rotating the coil in a strong magnetic field.

The primary coil P (Fig. 130) and the secondary S are wound upon an iron core C, which is mounted on an axle and driven by gearing from the petrol motor itself. The magnetic field is produced by a strong, permanent horseshoe magnet with pole pieces N and S. In the position shown in (*a*) the magnetic lines of force are entering at the end A of the core and leaving at B. In (*b*) the lines are entering at B and leaving at A. Hence, in the movement from one position to the other the lines have been reversed in the primary coil P, and an e.m.f. is produced. Owing to the low resistance of P, a large current will flow, and by means of a contact key this primary current is interrupted at the moment

that the spark is required to fire the explosive mixture of gases in the cylinder, the spark gap being in series with the secondary coil. Just as in the induction coil, a condenser in parallel with the spark increases the efficiency of the magneto. This condenser is mounted on the same axle as the core and the coils and rotates with them.

Self-induction.—After studying the inductive effect of one circuit upon another, it will be understood that when the current in any circuit is started, magnetic lines of force

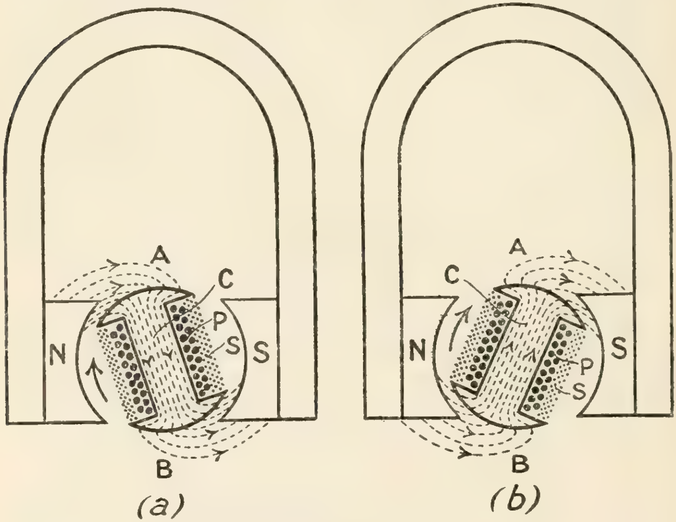


FIG. 130.—Circuits of the magneto.

become threaded with the circuit which were not there before the current began to flow. Now the rate at which magnetic lines of force enter a circuit is an electromotive force in the circuit (p. 141). Thus while the current in a circuit is growing, an e.m.f. acts in the circuit due to this growth, and as on page 144, it is opposite in direction to the current itself. Hence in Fig. 131 the arrow *a* represents the direction of this e.m.f. while the current *i* is growing. Similarly, when the current *i* decays there is an e.m.f. *b* in the direction of the current. Thus *a* causes the current to increase **more slowly**

than it otherwise would, and b causes it to decay **more slowly** than if there were no **self-induction.** It will therefore be seen that if the circuit has many turns, still more, if there is an iron core, the effect of self-induction may be very great, and the current will grow and decay much more slowly than if there is little self-induction.

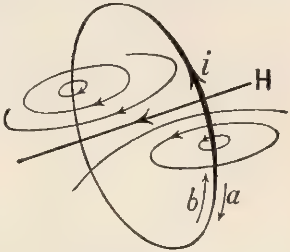


FIG. 131.—Circuit illustrating self-induction.

The **coefficient of self-induction,** or the **self-inductance** of a circuit is defined in a similar manner to the mutual inductance of two circuits (p. 145). It is the e.m.f. produced in the circuit when the current grows at unit rate, or

$$\text{Self-inductance} = \frac{\text{e.m.f. due to change of current}}{\text{rate of change of current}}$$

$$= \frac{\text{e.m.f.}}{\text{change of current per second}}$$

Self-inductance, like mutual inductance, may be measured in absolute C.G.S. units or in henries (p. 145).

Thus—

$$1 \text{ henry} = \frac{1 \text{ volt}}{1 \text{ ampere per second}}$$

and, $\quad 1 \text{ henry} = 10^9$ absolute units of inductance

It is due to the self-inductance of a circuit that a spark is produced whenever a circuit carrying a current is broken. For as the current decreases, the magnetic lines of force collapsing on the coil produce an electromotive force which tends to make the current continue. The greater the rate of decay of the current the greater is the e.m.f. due to self-inductance, and the current is enabled to jump the air gap. With big electro-magnets, the self-inductance may be several henries, and the spark on breaking the circuit becomes an arc, or if the ends of the conductors are held in the hands a severe shock will be produced.

Ordinary coils have self-inductance of small amount, being seldom more than a few millihenries.

Self-inductance of solenoid.—It is not difficult to calculate the self-inductance of a long solenoid, for the strength of

magnetic field inside the solenoid is $4\pi ni$, where n is the number of turns of wire per centimetre length of the solenoid and i the current flowing in it. If, then, the area of section of the solenoid is a sq. cm., the total number of magnetic lines of force passing through the solenoid is $4\pi nia$. Suppose that the current takes 1 second to cease, $4\pi nia$ lines are removed in 1 second, and the e.m.f. in each turn would be $4\pi nia$ C.G.S. units. But if the length of the solenoid is b cm., there are in all nb turns in series, and the e.m.f. in each is $4\pi nia$.

$\therefore$ total e.m.f. is $4\pi nia \times nb = 4\pi n^2 abi$

But,

$$\text{self-inductance} = \frac{\text{e.m.f.}}{\text{rate of change of current}}$$

$$= \frac{4\pi n^2 abi}{i}$$

$$= 4\pi n^2 ab \text{ C.G.S. units}$$

$$= 4\pi n^2 ab \times 10^{-9} \text{ henries}$$

It will be noticed that the same result would have been arrived at, whatever the time in which the current is supposed to decay. Also it is assumed that there is no iron inside the solenoid. The presence of iron renders the problem of inductance extremely complex.

Lenz's law.—The several laws of electrodynamics may be collected into one law of greater generality then either of the laws taken separately. This law is due to Lenz and has several useful applications. Lenz's law may be stated as follows :—

Whenever a conductor moves in a magnetic field, the currents induced in the conductor are in such a direction that the reaction between their magnetic fields and the original magnetic field opposes the motion of the conductor.

Lenz's law may be seen to be true in the case of a bar magnet NS (Fig. 132) approaching a coil A which has closed circuit. From the law on p. 141 it will be seen that the induced current in A flows in the direction shown by the arrows. This current has a magnetic field such that one side of the coil is a pole N′ and the other S′. The repulsion between N and N′ opposes the motion of the magnet. In a similar manner it may be shown that any relative motion of the coil and the magnet is opposed.

If a massive piece of metal of high conductivity, such as silver or copper, be moved about between the poles of an electro-magnet, the force opposing the motion may be easily felt. If the metal be suspended so that it can swing, any motion given to it will be destroyed quickly. For this reason the suspended coil of a galvanometer (p. 135) is wound upon a copper frame when it is desired that the coil shall be nearly dead-beat, that is, that it shall come to rest quickly For any motion of the coil is accompanied by induced currents in the copper frame, which, by Lenz's law, oppose the motion. For a similar reason, when the coil or needle oscillates to an extent which is troublesome, it may be brought to rest by short-circuiting the galvanometer so that the oscillation produces induced currents. It is clear that if the coil is on open circuit, or if the metal moved between the poles of an electromagnet be of poor conductivity, the induced currents will be absent or very small and the Lenz effect will be feeble.

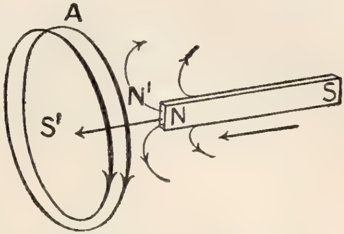


FIG. 132.—Illustration of Lenz's law.

Arago's rotation.—An experiment due to Arago in 1825 could not be explained at that time, but Faraday's discovery of induced e.m.f.'s gave the explanation. A pivoted magnet A (Fig. 133) is situated above a copper disc B which can be

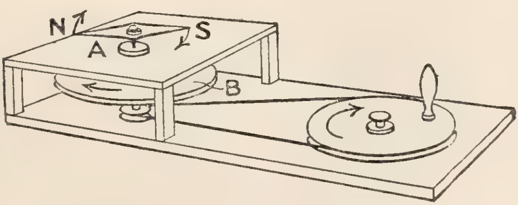


FIG. 133.—Arago's rotation.

caused to rotate. The magnet is protected from air currents caused by the disc, but will still be found to follow the rotation of the disc. The reason for this follows from Lenz's law, for the induced currents in the disc due to cutting the magnetic field of the magnet produce forces which

oppose the motion of the disc, or rather oppose the relative motion of disc and magnet. Hence the magnet follows the disc. If the disc had been suspended and the magnet rotated, the disc would then have followed the rotation of the magnet. In Arago's experiment the speed of the magnet can never equal that of the disc, for there would then be no relative motion of the two and no cutting of the lines of force, so that there would be no currents induced in the disc.

Exercises on Chapter IX

1. State the law of force on a current situated in a magnetic field.

A current of 15 amperes flows in a wire inclined at an angle of 30° to a magnetic field of strength 40 gauss. What is the force on each centimetre of the conductor?

2. Find an expression for the couple acting on a rectangular coil of wire carrying a current and situated in a magnetic field.

3. A rectangular coil 3 cm. × 2 cm. consisting of 30 turns is situated in a magnetic field of strength 20 with its plane making an angle of 20° with the field. If a current of $\frac{1}{10}$ ampere flows in the coil, find the couple acting upon it.

4. Describe some form of suspended coil galvanometer and point out its superiority to the suspended needle galvanometer.

5. State the law for the amount of work done in carrying a magnetic pole round an electric current, and find the strength of magnetic field near a long, straight current.

6. Two circles of diameter 18 cm. of copper wire are co-axial and their planes are $\frac{1}{2}$ mm. apart. If each carries a current of 2 amperes, find the force between the coils, assuming the law to be the same as for straight conductors.

7. Show that currents in the same direction attract each other, and currents in opposite directions repel each other. Describe some instrument in which this principle is used for measuring current.

8. Describe the principle of the watt-meter.

9. State the law of production of electromotive force in a conductor by cutting magnetic lines of force. What is the e.m.f. in a conductor 25 cm. long moving at right angles to itself and to a magnetic field of strenth 40, with velocity 10 metres per second.

10. A coil of area 80 sq. cm. consisting of 200 turns rotates in a magnetic field of strength 20 about an axis in its own plane, making 1,200 revolutions per minute. Find the average e.m.f. in the coil.

11. What is mutual inductance? A solenoid of 80 turns per cm. and cross-section 10 sq. cm. has wound upon its central part a coil of 1,000 turns. Find the mutual inductance of the coils.

12. Describe the induction coil. Why is the e.m.f. at the break of the primary current greater than that at the establishment of the primary current?

13. What is self-induction? Explain why the self-induction in a circuit affects the rate of growth and decay of the current.

14. Define the units of inductance and the relation of the practical units to the absolute units.

15. Calculate the self-inductance of a solenoid of 25 turns per cm., diameter 6 cm., and length 60 cm.

16. What is Lenz's law? Describe some application of it.

17. Calculate the inductance of a solenoid of length 30 cm. having 80 turns per centimetre, if the area of cross section of the solenoid is 7 sq. cm.

18. What is the henry, and how is it related to the absolute C.G.S. unit of inductance?

CHAPTER X

ALTERNATING CURRENTS

Rotating coil.—When a coil rotates in a magnetic field, the number of magnetic lines of force threaded through the coil is varying continually. In Fig. 134, when the coil ABCD

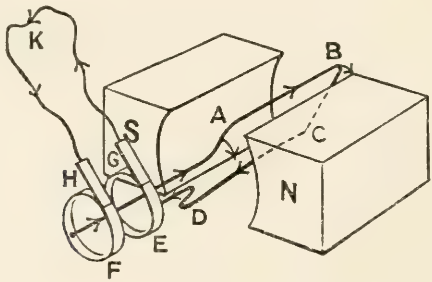


FIG. 134.—Simple alternator. Diagrammatic.

is vertical, magnetic lines of force from the poles NS pass through it. When the coil is horizontal no lines pass through it. Therefore, in passing from the vertical to the horizontal position, all the lines threaded through it have been removed, and the rules on pp. 140 and 141 show that the e.m.f. is in the direction ABCD round the coil. In the next quarter-revolution the lines are added, but from the opposite side of the coil, so that the e.m.f. is still in the direction ABCD. During the next half-revolution the e.m.f. is in the direction DCBA. If A is connected to a ring F, and D to E, and brushes H and G touch these rings, the brushes being connected by an external conductor HKG, then the e.m.f. in the coil will produce a current in the external circuit, which

will be reversed every half-revolution of the coil. Such a one is said to be an **alternating current,** and the e.m.f. in the coil is called an **alternating electromotive force.**

Alternating e.m.f.—In order to find the e.m.f. in the rotating coil at any instant, consider an end view of the coil as in Fig. 135. When the plane of the coil makes angle θ

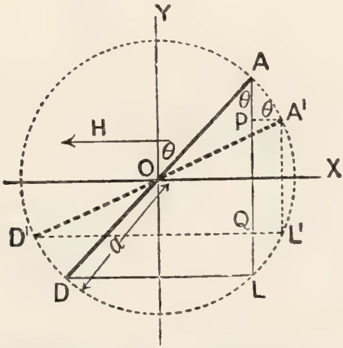


FIG. 135.—Diagram for finding e.m.f. in rotating coil.

with the vertical axis OY, the number of magnetic lines of force threaded through the coil is $H \times AL \times b$, where b is the length of the coil AB in Fig. 134. Now, when the coil has rotated through the further small angle θ' to the position A′D′ the number of lines threaded through the coil is $H \times A'L' \times b = H \times PQ \times b$. The number of lines cut out in the rotation θ' is therefore $Hb(AL - PQ) = 2HbAP$.

The time t occupied by the rotation θ' is found from the angular velocity of rotation ω.

For, $$\frac{AA'}{a} = \theta'$$

where a is the radius AO,

and, $$\frac{\theta'}{t} = \omega$$

$$\therefore\ t = \frac{AA'}{a\omega}$$

$$\text{Now, e.m.f. in coil} = \frac{\text{number of lines of force removed}}{\text{time}}$$

$$= \frac{2\text{H}b\,.\,\text{AP}}{\dfrac{\text{AA}'}{a\omega}}$$

$$= 2ab\,.\,\text{H}\,.\,\omega\,.\,\frac{\text{AP}}{\text{AA}'}$$

$$= \text{area of coil} \times \text{H}\omega\,.\,\frac{\text{AP}}{\text{AA}'}$$

If now the angle θ' is very small so that the e.m.f. is the instantaneous value, AP/AA$'$ is $\sin\theta$, in which case—

$$\textbf{instantaneous e.m.f.} = \textbf{AH}\,\omega\,\textbf{sin}\,\theta$$

where A is now written for the area of the coil. It is, of course, the effective area, which may include any number of turns.

When the coil is vertical, $\theta=0$ and $\sin\theta=0$, so that the e.m.f. is zero. The maximum e.m.f. occurs when $\theta=90°$ and $\sin\theta=1$. The value of this maximum e.m.f. is therefore AHω, which is the e.m.f. when the coil is parallel to the magnetic field.

As an example, consider a coil of 100 turns, each of area 600 sq. cm., making 1,200 revolutions per minute in a magnetic field of strength 30 gauss.

$$\text{H} = 30, \qquad \text{A} = 100 \times 600 = 60{,}000 \text{ sq. cm.}$$

$$\omega = \frac{1200}{60} \times 2\pi = 40\pi \text{ radians per second}$$

$$\therefore \text{ maximum e.m.f.} = 60{,}000 \times 30 \times 40 \times \pi$$
$$= 2{\cdot}262 \times 10^8 \text{ absolute units}$$
$$= 2{\cdot}262 \text{ volts}$$

If we write E_0 for the maximum e.m.f. in the coil, then $E_0 = AH\omega \times 10^{-8}$ volts, and the instantaneous e.m.f. E at any instant is given by—

$$E = E_0 \sin\theta$$

On plotting E against θ, the curve shown in Fig. 136 is

obtained, which is a sine curve and shows how the e.m.f. changes in an alternating current circuit.

The simple sine form for the alternating e.m.f. is seldom attained, because, with a simple system as in Fig. 134, the magnetic

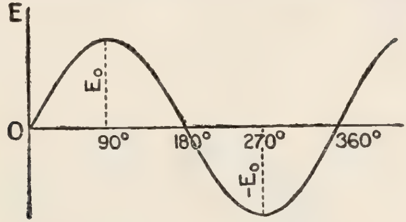


FIG. 136.—Alternating e.m.f.

field is not uniform, and in an actual alternator the coils themselves are not of simple form. Nevertheless, although not strictly sine in form, the alternating e.m.f.'s used in practice approximate to this simple type.

Alternators.—In using alternating current for lighting purposes it is necessary that a fairly large number of alternations per second should be attained, or the light will flicker sensibly. The lowest frequency used in practice is 50 cycles per second. If a single coil were used this would mean 50 revolutions per second or 3,000 per minute, a very high speed. Therefore to obtain the desired number of cycles per second, more coils are used. But this necessitates using more pairs of magnetic poles.

A diagrammatic representation of the alternator is given in Fig. 137. In this case there are six pairs of magnetic poles NS, and the magnetic field is excited by a direct current in the coils shown in thin line. The coils of the alternator, ABC . . . KL, are shown in thick line, and the circuit is completed through the slip rings PQ and the external circuit R. It will be seen that in the position shown, each coil is approaching a pole. But alternate poles are of opposite kind so that the e.m.f.'s in the coils are oppositely directed. This is allowed for by winding the coils in opposite directions, so that in the circuit the e.m.f.'s are in the same direction at each instant. The e.m.f. is alternating because of the reversal in sign as each coil passes a pole. In order to magnetise the iron poles, the direct current necessary must be produced by a small direct-current dynamo. The e.m.f. of the

alternator can be regulated by a rheostat placed in the direct-current circuit.

It is the relative motion of the coils and the magnetic poles which produces the e.m.f. in the alternator. It does not matter which are fixed and which move. The commoner arrangement is to have the poles fixed and the coils rotating, but in some cases the coils are fixed and the magnets rotate. In this case, of course, the slip rings must be in the direct-current circuit, and the alter-

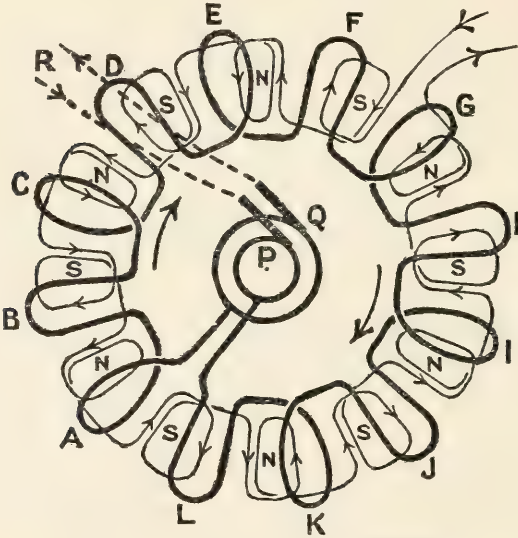


FIG. 137.—Circuits of 12-pole alternator.

nating circuit, being stationary, does not require slip rings. Whichever of the two is fixed is called the **stator.** The rotating part is called the **rotor,** and it is clear that rotor and stator are interchangeable.

Transformers.—The great advantage of the alternating over the direct current lies in the fact that it can be changed from one voltage to another with very little loss of energy. Thus it can be carried over long distances at high voltage and small current, only small copper mains being necessary, and transformed to low voltage and large current at the point at which it is to be used. The apparatus for effecting the change is called a **transformer**. The transformer

resembles the induction coil (Fig. 129), in fact it is developed from the induction coil. It consists of two coils of wire wound upon the same iron core. The supply current passes through the **primary** coil and the current to be used is obtained from the **secondary** coil. For transforming from low to high voltage the primary coil consists of few turns of thick wire and the secondary coil of many turns of thin wire. Such a one is a **step-up** transformer. A **step-down** transformer has many turns for the primary coil and few for the secondary.

The theory of the transformer is complicated, but considered simply we may note that the iron core is magnetised by the primary current. The magnetisation of the core is therefore alternating, and as the magnetic lines of force cut the secondary they produce an e.m.f. in it. The lines must also cut the primary coil, and with negligible resistance, the e.m.f.'s in the primary and secondary coils are proportional to the number of turns in each, since the same number of magnetic lines cut both coils. Thus as a first approximation—

$$\frac{\text{e.m.f. in secondary}}{\text{e.m.f. in primary}} = \frac{\text{number of turns in secondary}}{\text{number of turns in primary}}$$

A transformer is represented in Fig. 138. The primary and secondary turns are shown wound upon a laminated iron core.

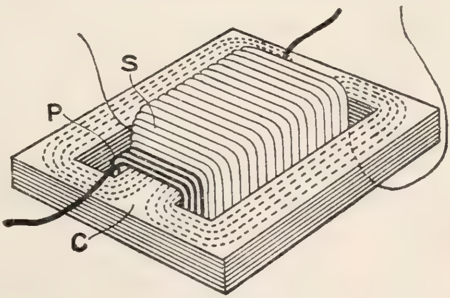


FIG. 138.—Transformer.

The iron circuit is always completed outside the coils, or the efficiency of the transformer would be reduced. In this complete circuit there are no free magnetic poles. Such poles would reduce the magnetisation of the core (p. 231) and hence the e.m.f. in the secondary is smaller than would otherwise be the case.

By representing the primary current as a sine curve ABCDEF

(Fig. 139), and considering that the magnetisation of the iron core is proportional to the primary current, it follows that where this varies most rapidly, as at A, C and E, the secondary e.m.f. will be

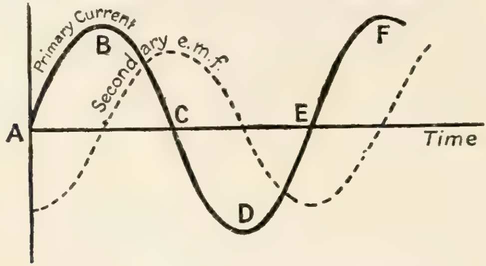


FIG. 139.—Current and e.m.f. curves.

greatest. Similarly at B, D and F, when the primary current is constant for an instant, the secondary e.m.f. is zero. The dotted curve indicates the manner in which the secondary e.m.f. varies.

Alternating current circuit.—In the case of a direct or constant current, the relation $\frac{E}{R} = I$, derived from Ohm's law, enables us to calculate the current whenever the e.m.f. of the circuit and its resistance are known. When, however, the current is changing, as in an alternating circuit, this simple equation is no longer sufficient for the calculation of the current. For, the inductance in the circuit affects the rate of change of current (p. 149) and if there is capacity (p. 54) this too will influence the current.

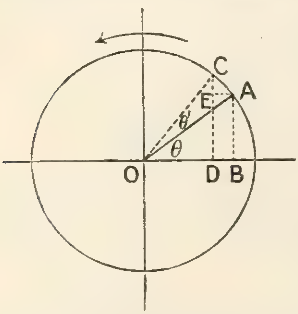


FIG. 140.—Diagram for alternating current.

A method similar to that on p. 155 will enable us to find the relation between current and e.m.f. when the circuit has inductance as well as resistance. The current being a quantity which varies as the **sine** of an angle is an example of a quantity which varies in a **simple harmonic** manner and may be represented by the projection of a rotating line upon a fixed line. Thus if OA (Fig. 140)

represents the maximum current I_0, $AB = I_0 \sin \theta$, represents the instantaneous current I—

$$\therefore\ I = I_0 \sin \theta$$

If OA rotates with angular velocity ω, about O, I represents an alternating current.

After a very short interval of time t, let OA have rotated through the small angle θ', and take the position OC. The current is then CD and the increase in current is $CD - AB = CE$.

From p. 149 we see that the inductance L is the e.m.f. due to unit rate of change of current,

$$\therefore\ \text{e.m.f. due to change in current} = L \times \frac{CE}{t}$$

But. $$\omega = \frac{\theta'}{t}, \quad \text{and,} \quad \frac{AC}{OA} = \theta'$$

$$\therefore\ t = \frac{AC}{\omega \,.\, OA} = \frac{AC}{\omega I_0}$$

$$\therefore\ \text{e.m.f. due to change in current} = L\omega I_0 \,.\, \frac{CE}{AC}$$

But if θ' is very small, $$\frac{CE}{AC} = \cos \theta$$

$$\therefore\ \text{e.m.f. due to change in current} = L\omega I_0 \cos \theta$$

Now the e.m.f. used in overcoming the resistance R in the circuit $= RI = RI_0 \sin \theta$.

Therefore if E is the instantaneous value of the e.m.f. in the circuit—

$$E = L\omega I_0 \cos \theta + RI_0 \sin \theta$$

E itself being an alternating e.m.f. may be represented by a rotating line, but it does not follow that it is in the same direction as the line representing I. As the effect of inductance is to oppose the growth of the current (p. 149) it is reasonable to suppose that the current would lag behind the e.m.f., or, what is the same thing, the e.m.f. would lead the current by an angle, say, θ_0.

Then, $$E = E_0 \sin (\theta + \theta_0)$$

and the above equation becomes—

$$E_0 \sin (\theta + \theta_0) = L\omega I_0 \cos \theta + RI_0 \sin \theta$$

We see that this condition is fulfilled if we draw $OA=E_0$ (Fig. 141) and OB making an angle $AOB=\theta_0$ behind OA. Upon OB drop a perpendicular AB, and drop perpendiculars AE and BC upon OX.

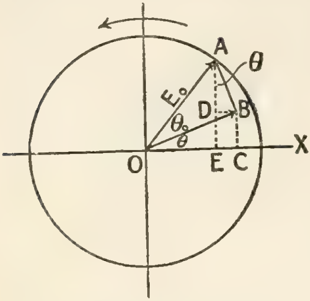


FIG. 141.—E.m.f diagram for alternating current circuit.

Then, $AE=AD+DE$
$=AD+BC$

Now, $AE=E_0 \sin(\theta+\theta_0)$
$BC=OB \sin\theta$

and, $AD=AB\cos\theta$

$\therefore E_0 \sin(\theta+\theta_0)$
$=AB\cos\theta+OB\sin\theta$

Comparing this with—

$E_0\sin(\theta+\theta_0)$
$=L\omega I_0\cos\theta+RI_0\sin\theta$

we identify AB as $L\omega I_0$, and OB as RI_0.

Further, from the right-angled triangle OAB—

$$OA^2 = AB^2+OB^2$$

$$\therefore\ E_0{}^2 = L^2\omega^2 I_0{}^2+R^2 I_0{}^2$$

$$\therefore\ I_0 = \frac{E_0}{\sqrt{L^2\omega^2+R^2}}$$

This gives the relation between current and e.m.f. with inductance L, resistance R, and ω which determines the rate of alternation.

Further, $$\tan\theta_0 = \frac{L\omega}{R}$$

therefore the angle θ_0 of lag of the current behind the e.m.f. is such that its tangent is $\frac{L\omega}{R}$.

Also, $$I=I_0\sin\theta = \frac{E_0}{\sqrt{L^2\omega^2+R^2}}\sin\theta$$

If now we reckon time t, in seconds from the instant at which the rotating line OA representing E_0 was in the direction OX, the angle $AOX=\omega t$, and $\theta=\omega t-\theta_0$,

$$\therefore\ E=E_0\sin\omega t$$

and, $$I=\frac{E_0}{\sqrt{L^2\omega^2+R^2}}\sin(\omega t-\theta_0)$$

The angle ωt is called the **phase** of the e.m.f. and $(\omega t-\theta_0)$ is the phase of the current.

It is now possible to draw the e.m.f. and current curves in any given case; for, the line E_0 (Fig. 142 (*a*)) representing the maximum applied e.m.f., I_0 is drawn at angle $\theta_0=\tan^{-1}\frac{L\omega}{R}$ behind E_0, and made equal in scale to $\frac{E_0}{\sqrt{L^2\omega^2+R^2}}$. Then for each position of E_0 and I_0, their vertical projections in Fig. 142 (*b*) represent the instantaneous values of the e.m.f. and current.

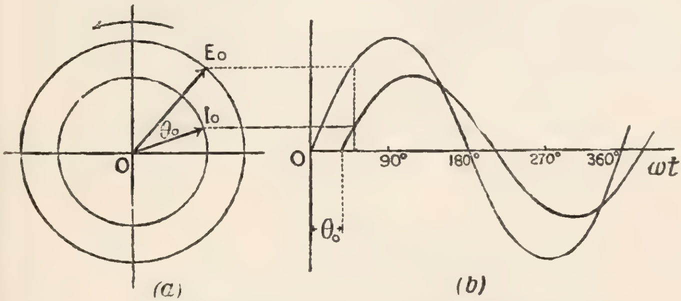


FIG. 142.—E.m.f. and current curves for alternating current circuit

The quantity $\sqrt{L^2\omega^2+R^2}$ plays a part in alternating current calculations similar to the resistance R in direct or continuous current problems. It is called the **impedance** of the circuit. Also the quantity $L\omega$ is called the **reactance.** Thus—

$$(\text{Impedance})^2=(\text{Reactance})^2+(\text{Resistance})^2$$

Example.—An alternating e.m.f. of 100 volts and 50 cycles per second is applied to a circuit of inductance 0·02 henry and resistance 4 ohms. Find the maximum current and its lag in phase behind the e.m.f.

Here, $\omega=50\times 2\pi$

since one revolution is an angle of 2π radians,

$$\therefore \text{ impedance} = \sqrt{(0{\cdot}02\times 50\times 2\pi)^2+4^2}$$
$$= \sqrt{(2\pi)^2+16}$$
$$= 7{\cdot}447$$

$$\therefore\ I_0 = \frac{E_0}{\sqrt{L^2\omega^2+R^2}} = \frac{100}{7{\cdot}447}$$
$$= 13{\cdot}43 \text{ amperes}$$
$$\tan\theta_0 = \frac{L\omega}{R} = \frac{0{\cdot}02\times 50\times 2\pi}{4} = \frac{6{\cdot}282}{4}$$
$$= 1{\cdot}570$$
$$\therefore\ \theta = 57^\circ\ 30'$$

That is, the current lags 57° 30′ behind the e.m.f.
The equation to the current is therefore—

$$I = 13{\cdot}43 \sin(314{\cdot}1t - 57^\circ\ 30')$$

Circuit with capacity inductance and resistance.—In this case the treatment is more complicated than in the previous one, but the result is of interest—

$$I = \frac{E_0}{\sqrt{\left(\frac{1}{C\omega} - L\omega\right)^2 + R^2}} \sin(\omega t + \theta_0)$$

and,
$$\tan\theta_0 = \frac{\frac{1}{C\omega} - L\omega}{R}$$

Thus, the impedance is $\sqrt{\left(\frac{1}{C\omega} - L\omega\right)^2 + R^2}$

Measurement of alternating current.—It has already been pointed out that the suspended-coil galvanometer or ammeter is useless in an alternating current circuit. The instrument reads the average value of the current when the current changes rapidly. But in the case of an alternating current the average value taken over any appreciable time is obviously zero. Hence the instrument on such a circuit always reads zero. If, however, we consider the soft iron ammeter (p. 90) or the Kelvin current balance (p. 138) we find that the reading of these instruments depends upon the **square of the current.** Now the square of a quantity never changes sign; it is always positive, so the reading of the instrument is always in the same direction, whatever the direction of the current, and the instrument gives a reading when an alternating current flows through it.

The reading obtained for any given alternating current must be found by obtaining the **average value of the square of the**

current, since the reading of the instrument depends upon the square of the current. In Fig. 143 a complete cycle for an alternating current of maximum value I_0 is plotted, and the curve of I^2 is also plotted, by squaring each value of the current. This curve has the maximum value I_0^2, and it can be shown by analysis or by testing with tables that this is also a sine curve, but with axis AB, where $OA = \frac{I_0^2}{2}$. Thus the curve or straight line AB has the same average value as the curve I^2, and the continuous current of value $\frac{I_0}{\sqrt{2}}$ would therefore have the same average value for

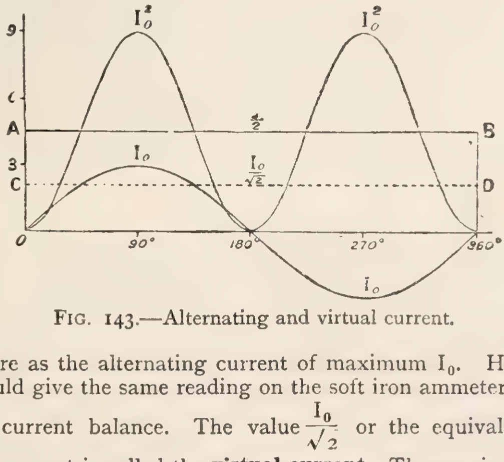


FIG. 143.—Alternating and virtual current.

its square as the alternating current of maximum I_0. Hence the two would give the same reading on the soft iron ammeter and the Kelvin current balance. The value $\frac{I_0}{\sqrt{2}}$ or the equivalent continuous current is called the **virtual current.** Thus an instrument calibrated by means of continuous current, reads virtual current or virtual amperes on alternating current. For alternating current—

$$\text{Virtual current} = \frac{\text{maximum current}}{\sqrt{2}}$$

when the current is a true sine current. In any case it is the virtual current which it is desired to know in practice, not the maximum value.

In a similar manner, an instrument which gives a reading proportional to the square of the e.m.f. reads **virtual volts.** Such an instrument is the quadrant electrometer, when connected as described on p. 68. Many such instruments are in use, in which the desired sensitiveness is obtained by

having many sets of quadrants one above the other. Such an instrument is called a **multicellular** electrostatic voltmeter.

The virtual volts and virtual amperes are the all-important quantities, the maximum values are seldom required. Thus the heating in a circuit is proportional to the square of the current, and therefore to the square of the virtual amperes. Similarly the power in watts expended in a circuit is equal to the product of virtual volts and virtual amperes when there is no lag in current behind e.m.f. If there should be such a lag the power is—

$$(\text{Virtual volts}) \times (\text{Virtual amperes}) \times \cos \theta_0$$

in watts, where θ_0 is the lag of current behind e.m.f. The product (virtual volts) $\times$ (virtual amperes) is called the **apparent watts,** and the ratio of true watts to apparent watts is called the **power factor** of the circuit. Thus—

$$\frac{\text{True watts}}{\text{Apparent watts}} = \text{power factor}$$

The power factor is, of course, unity when there is no lag of current behind e.m.f.

Exercises on Chapter X

1. Find an expression for the e.m.f. produced in a coil rotating in a magnetic field about an axis perpendicular to the field.

2. A coil of 30 turns of area 80 sq. cm. per turn makes 2,000 revolutions per minute about an axis at right angles to a magnetic field of strength 60 gauss. Find the maximum e.m.f. produced in the coil, and the e.m.f. when the plane of the coil makes an angle of 45° with the magnetic field.

3. Explain the method of representing an alternating e.m.f. by means of a sine curve.

4. Describe some form of alternator, pointing out its essential parts.

5. Describe some form of transformer and explain its use.

6. Find a value for the current in a circuit having inductance and resistance to which an alternating e.m.f. is applied.

7. Calculate the maximum current in a circuit of inductance 0·5 henry and resistance 8 ohms to which an alternating e.m.f. of maximum value 150 volts with frequency 60 cycles per second is applied.

8. Explain the terms "impedance" and "reactance."

9. Explain why the current lags in phase behind the e.m.f. in an alternating current circuit having inductance and resistance. What is the value of the lag in any given case?

10. Explain the meaning of "virtual" current. What is the value of the virtual current if the maximum current is 50 amperes?

11. An alternating current at 200 volts and 50 cycles per second is applied to a circuit of resistance 2 ohms. What inductance must be placed in the circuit if the current is to be 50 amperes?

12. Find the lag of current behind e.m.f. in a circuit in which the true watts and apparent watts are 500 and 600 respectively.

CHAPTER XI

DIRECT CURRENTS

Simple dynamo.—Any machine, driven by mechanical means, which produces electric current may be called a **dynamo,** but it is customary to apply the term to a machine for producing direct current, that for producing alternating current being called an alternator. The simplest form of direct current dynamo is a single rotating coil ABCD (Fig. 144) which may have many turns, although, for

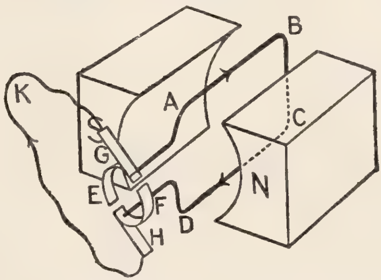


FIG. 144.—Simple direct-current dynamo.

simplicity, only one is shown in the diagram. It was seen on p. 154 that when such a coil rotates, the e.m.f. in it is alternating. If, therefore, the current is required to be always in the same direction in the external circuit, some form of **commutator** is necessary. This consists of the ring EF, divided into two parts which are insulated from each other. To the part E the end A of the coil is connected, and to F D is connected. The brushes G and H bear upon the ring, touching it at opposite ends of a diameter. It follows that the descending side of the coil is always connected to G and the ascending side to H, and the current in the external circuit is always in the direction HKF.

Although constant in direction, the current in the external circuit is by no means constant in value. In the external circuit the e.m.f. fluctuates. Referring again to Fig. 136, it is seen that the e.m.f. for the rotating coil gives a sine curve. The effect of the commutator is to reverse the connections of coil and external circuit twice in each revolution. The e.m.f. acting in the external circuit will therefore fluctuate as shown in Fig. 145. This is a serious objection, and the e.m.f. must be made much more nearly constant. This is attained by arranging coils inclined at various angles, so that the maxima of e.m.f. are not all attained at the same time. Thus with two coils at right angles, the dotted curves in Fig. 146 indicate the e.m.f.'s in each coil, and if the coils are in series, the resultant e.m.f. is obtained by adding the e.m.f.'s in the two coils. By adding more and more coils at the intermediate angles, the unevenness of the resultant e.m.f. curve may be nearly smoothed out.

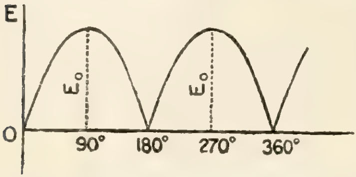


FIG. 145.—Current in simple dynamo.

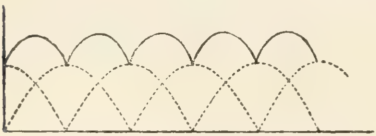


FIG. 146.—Current in dynamo having two coils.

Direct-current dynamo.—The method of connecting the conductors in series has many varieties. The conductors with the iron core to which they are fixed is called the **armature**, and fixed to the same axle as the armature and rotating with it is the **commutator.** In Fig. 147 is seen an armature having 16 conductors let into the longitudinal slots in the iron core, and a commutator of eight segments. The electromagnet NS, known as the **field magnet**, together with the armature core, produces an intense magnetic field in the space through which the armature conductors move. With the arrangement shown, the e.m.f. in the armature conductors on the right-hand or descending side is from back to front, and in those on the left-hand or ascending side, from front to back. The connections between the armature conductors at the front are shown in full line and those at the back in dotted line. The current enters the armature by the brush

C, and by tracing the connections it will be seen that there are two paths open to it, but both paths lead to the brush D by which the current leaves and goes by way of F to the external circuit. With the arrangement shown, the main current also passes through the **field coils** A and B, which excite the field magnets. In very small machines the magnetic field is sometimes produced by permanent magnets

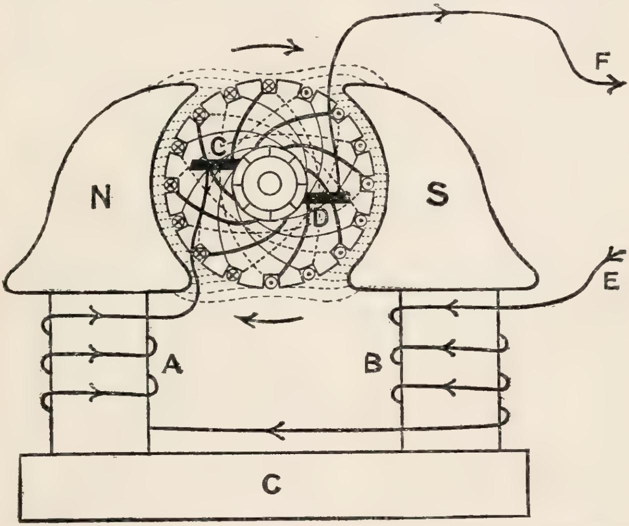


FIG. 147.—Diagrammatic representation of the electrical circuits of a direct-current series-wound dynamo.

of the type seen in Fig. 130, but in all large dynamos current-excited field magnets are used. When the field coils are in series with the armature and external circuit the machine is said to be **series wound,** and is a **series dynamo.**

There are several objections to a series dynamo, one of the chief being that it must not be used for charging accumulators. For, if the e.m.f. of the machine should, for any reason fall below that of the battery, the current is reversed and the polarity of the machine is also reversed. This will mean that the machine is helping to *discharge* the battery and is being driven as a motor (p. 171).

The **shunt-wound dynamo** is not open to the same objection. In this case the field coils are in parallel or shunt with the external circuit. The current from the brush D (Fig. 148) divides, part going through the external circuit EF and the rest through the field coils AB. It will then be seen that if the current in the external circuit be reversed, that in the field coils will not be reversed and the magnetisation of the machine will be maintained of correct polarity.

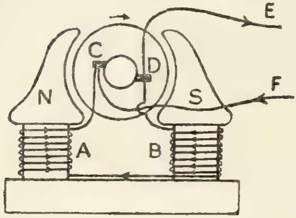


FIG. 148.—Shunt-wound dynamo.

Characteristic of dynamo.—In forming an opinion upon the behaviour of a dynamo it is customary to run it at constant speed and take a number of readings of the current and the potential difference between its terminals. On plotting one of these against the other a curve is obtained which is called the **characteristic** of the dynamo. The characteristics of the series and the shunt machines differ essentially in form. Whereas for the series dynamo the characteristic has the form OAB (Fig. 149), since the magnetisation of the field magnets is due to the main current, that of the shunt dynamo has the form CDE, since the p.d. drops with increasing current. The reason for this is that the increase in current is due to a decrease in resistance in the external circuit, and therefore a smaller current flows in the shunt circuit (see p. 78). If for example the external resistance were reduced to zero, there would be no current in the shunt coils and the field magnets would not be excited at all.

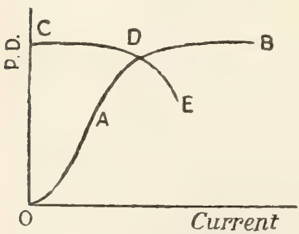


FIG. 149.—Characteristics of dynamos.

By combining the series and shunt arrangements, a dynamo may be produced which maintains constant p.d. for all currents. This is called a **compound-wound dynamo.** It is usual to add a few series turns to the shunt turns, so that the drop in p.d. corre-

sponding to the shunt characteristics CD (Fig. 149) is made up for by the increased magnetisation due to the current in the series turns.

Electric motors.—Since the two laws of electrodynamics (pp. 133 and 140) are the converse of each other, so are the machines derived from them related in a converse manner. In the dynamo the armature is mechanically driven and the armature conductors cutting the magnetic field have e.m.f. and current produced in them. If then current from some external source be passed through a dynamo, the armature conductors in the magnetic field will experience a force and the armature will be driven round. On referring again to Fig. 147, if the current be caused to flow as shown, the conductors on the right-hand side of the armature will experience forces driving them upwards (p. 133) and those on the left-hand side will be driven downwards. Thus the machine will run as a motor in the **opposite direction** to that in which it was driven as a dynamo. If used in this way the brushes must, of course, be set at the appropriate angle.

It should be noticed that reversal of the current will not reverse the direction of rotation, because the reversal of current will reverse the polarity of the field magnets so that the forces will still be in the old direction. In order to reverse the direction of running of a motor, the connections to the field coils must be changed so that the polarity is reversed without reversing the current in the armature. The same considerations apply to the shunt-wound motor. In Fig. 150 is seen a typical electro-motor, but in the modern form of motor the machine is so cased in that its working parts cannot very well be seen.

Back e.m.f. in motor.—The resistance of the armature is generally so small that if the voltage of supply were applied while the motor were at rest the current would be so great that injury due to overheating would result. But when the motor is running, the armature conductors are cutting across magnetic field, and an e.m.f. is developed in them. This is called a **back e.m.f.** because it always opposes the current which is driving the motor. This fact may be proved, either by tracing out the direction of the e.m.f. from Fig. 147, or on general grounds. For if it did not oppose the current it must help it, and if this were the case it would be possible to start the motor and then it would run itself. This is contrary to universal experience.

Thus if the e.m.f. applied to the armature from an external

source is E, and the back e.m.f. is E′, and R is the resistance of the armature, then the current is given by—

$$I = \frac{E - E'}{R}$$

As the speed of the motor increases E′ gets greater and therefore the current drops. In order to keep the current to a safe amount when starting, it is customary to employ a **starting**

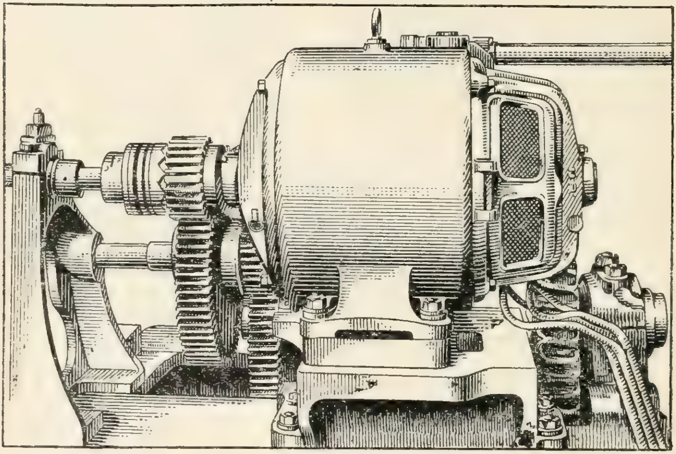


FIG. 150.—Electro-motor.

resistance. This resistance can be removed step by step as the speed increases, until at full speed the starting resistance is entirely removed, since the back e.m.f. then keeps the current down to a safe value.

Another device, used largely on electric trams and trains, is to arrange the motors in groups so that several are in series at starting, as in Fig. 151 (*a*). The combined resistance then ensures a reasonable value of the current. When the speed has increased somewhat, a switch changes the arrangement to (*b*), in which there are two groups of two motors in series across the mains. At higher speed the grouping is again altered to (*c*), each motor being connected directly across the mains.

Motors as brakes.—A further example of the reversibility of the laws of electrodynamics may be seen in the use of the motors themselves as brakes on electric trams and trains. If, when running, the motors be detached from the mains and then short-circuited, they act as dynamos and the direction

of current produced in the armature gives forces opposed to the direction of running (p. 171) so that a braking effect is produced. This effect is an excellent application of Lenz's law (p. 150). Of course a resistance must be introduced if the motors are running at full speed, or the current in them would be excessive, and the resistance reduced step by step as the speed decreases. Or, a grouping similar to that used at

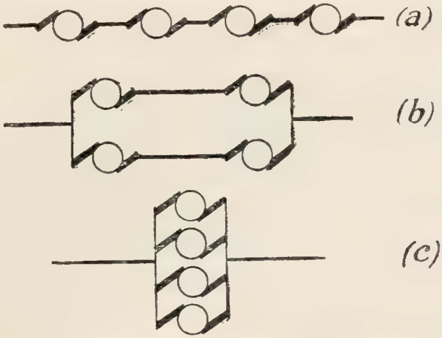


Fig. 151.—Traction. Arrangement of motors for starting.

starting (Fig. 151) may be employed. It must be noticed that such braking effect of the motors will not hold the train at rest upon an incline, for when the motors are at rest there is no current in them and therefore no action as a brake. A friction brake must then be employed.

Exercises on Chapter XI

1. Describe a simple form of direct-current dynamo and the method of commutation.

2. Explain how the dynamo may be constructed to give an electromotive force of nearly constant value.

3. Describe the series, shunt and compound methods of excitation of dynamos.

4. Give the forms of the characteristics of a series and a shunt dynamo, and account for these forms.

5. Explain why the dynamo will run as a motor. How can the direction of rotation of an electro-motor be reversed?

6. What is back e.m.f. in the case of an electro-motor? How are the difficulties encountered when starting a motor overcome?

7. Describe the principle of the use of motors as brakes.

8. If the magnetic field of a dynamo has average strength 500 gauss and effective area 600 sq. cm. and in each section of the armature there are 32 conductors in series, what is the e.m.f. of the machine when running at 1800 revolutions per minute?

CHAPTER XII

ELECTRIC WAVES AND RADIATION

Electric lines of force in motion.—Problems in electrostatics have been elucidated by means of electric lines of force (Chapter IV) and problems in magnetism by means of magnetic lines of force (Chapter II), but this is quite an artificial separation of the two sets of phenomena due to their origin in widely separated discoveries. It is due to the work of Sir J. J. Thomson that we may now make use of one set of lines of force (or induction) in dealing with both electric and magnetic fields.

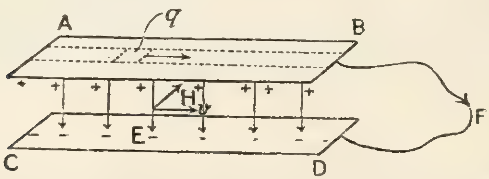


FIG. 152.—Relation between electric and magnetic fields.

Consider two parallel conducting strips AB and CD (Fig. 152) and let AB have a charge of q units of positive electricity per square centimetre, and CD q units of negative electricity per square centimetre. If the plates are of considerable extent in comparison with their distance apart, they are equivalent to a condenser, having capacity $1/4\pi t$ per unit area (p. 56). If E is the electric field between the plates, it is the force on unit positive charge placed between them, and the work done in carrying this unit charge from CD to AB is Et ergs, which is therefore the difference of potential V between the plates (p. 41).

$$\therefore \text{ Capacity per sq. cm.} = \frac{q}{V} = \frac{q}{Et}$$

$$= \frac{1}{4\pi t}$$

$$\therefore \frac{1}{4\pi t} = \frac{q}{Et}$$

or,
$$E = 4\pi q$$

That is, electric field between the plates is equal to 4π times the charge per square centimetre upon either plate.

So far the charges have been considered to be at rest. Now let the plates be joined by a wire BFD. A current will now flow from AB to CD. There will be a motion of positive charge to the right along AB and negative charge to the right along CD, both motions corresponding to a current in the direction ABFDC. Also the electric lines of force move to the right in the space between AB and CD. The current flowing along a strip 1 cm. wide is qv, where v is the velocity of the positive charge along AB or negative charge along CD. The current qv is given, of course, in electrostatic units, but in electromagnetic units it will be $\frac{qv}{3 \times 10^{10}}$ (p. 71). The whole current in AB is $\frac{qvb}{3 \times 10^{10}}$, where b is the width of the plate, and if a unit magnetic pole be carried once round this current, the work done is $\frac{4\pi qvb}{3 \times 10^{10}}$ (p. 136). But if H is the magnetic field between the plates, Hb is the work done on the pole,

$$\therefore \frac{4\pi qvb}{3 \times 10^{10}} = Hb$$

that is,
$$4\pi \times \frac{qv}{3 \times 10^{10}} = H$$

We have, therefore, two equations for q—

$$q = \frac{E}{4\pi}, \text{ and, } q = \frac{H \times 3 \times 10^{10}}{4\pi v}$$

$$\therefore H = \frac{Ev}{3 \times 10^{10}}$$

This magnetic field is parallel to the plates and at right angles to the direction of the electric field. We may therefore consider that **the motion of the electric lines of force at right angles to their length constitutes a magnetic field at right angles both to the direction of electric field and to the direction of motion.**

This saves trouble in dealing with cases of moving fields, for only one set or lines of force need be drawn. When they are at rest there is no magnetic field. When they are in motion the magnetic field is implied by this motion and its strength is given by the equation on p. 175.

Oscillatory discharge of condenser.—It has been considered up to now that on connecting the plates of a charged condenser, the condenser is discharged without anything further happening. It was shown, however, by Lord Kelvin that if the plates are connected by a wire in the form of a coil so that there is considerable magnetic field produced by the discharging current, then the process of discharge is not so simple.

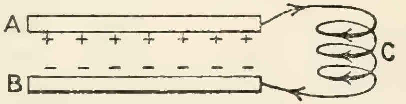


FIG. 153.—Charged plates of a condenser connected by a wire.

Consider the plates A and B (Fig. 153) connected by a coil of wire C. Then the discharge will begin, giving a current as shown. This current has a magnetic field and by the time the condenser is just discharged, the current has acquired maximum value, and the state of affairs is that shown in

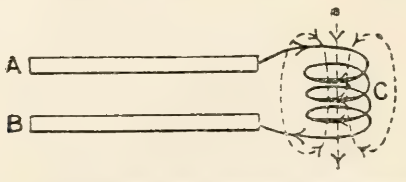


FIG. 154—Condenser at the moment when discharge is complete, but current is flowing in wire connection.

Fig. 154. The current will now begin to die away, but in doing so the magnetic lines of force collapse upon the coil and produce an e.m.f. in it, which makes the current continue (p. 149), which means that B will now acquire a positive charge and A a negative charge. This goes on until the state shown in Fig. 155 is reached. That is, the current has

ceased but the charges upon the plates are the reverse of the original charges. The process now starts in the opposite direction, and if it were not for loss of energy due to the

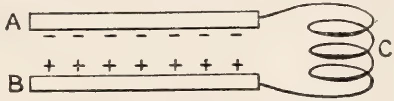


FIG. 155.—Condenser on complete reversal of charge.

current heating the wire, and any possible radiation of energy, the oscillation of charge would go on indefinitely, like the swinging of a pendulum when there is no friction.

It was shown by Lord Kelvin that the time of oscillation (T) of the charge is given by—

$$T = 2\pi\sqrt{LC} \text{ seconds}$$

where L is the inductance of the coil and C the capacity of the condenser, provided that the resistance of the circuit is not very great.

Or, if n is the number of oscillations per second—

$$n = \frac{1}{2\pi\sqrt{LC}}$$

The oscillatory nature of the discharge may be illustrated by placing a condenser, such as a Leyden jar, across the sparking

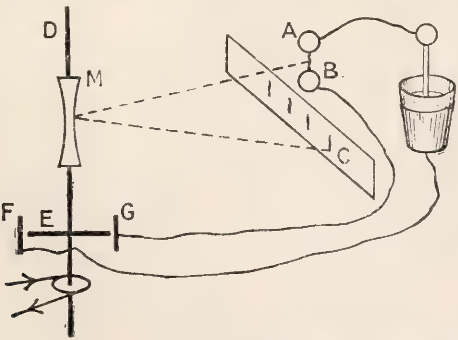


FIG. 156—Apparatus for demonstrating the oscillatory character of the discharge of a condenser.

knobs of the induction coil and examining the spark by means of a rotating mirror. A concave mirror M (Fig. 156) produces an

image of the spark AB upon the screen at C. On rotating the mirror rapidly about the axle D, using the contact EFG to produce the discharge at the proper moment, the image C will consist of one bright line, when the knobs AB are separated so that the resistance of the circuit is sufficient to prevent the oscillation of the discharge. But on approaching A towards B, the resistance of the circuit gets less, and when oscillations begin, the image C will be doubled at each discharge. On approaching the knobs still more, three or four images may be seen, showing that several oscillations occur at each spark before it is entirely quenched.

Electromagnetic waves.—From the equations of the electric field, James Clerk Maxwell calculated that any disturbance in the field would be propagated outwards with a velocity of 3×10^{10} cm. per second, the velocity of light, but it remained to Hertz to show by experiment the presence

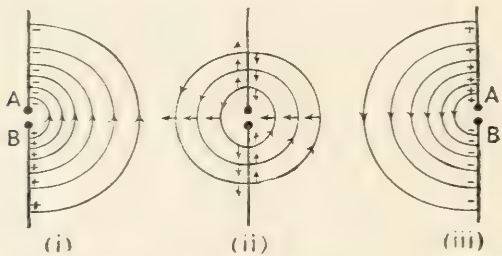


FIG. 157.—Electric lines of force surrounding an oscillator.

of these waves. Consider two wire conductors A and B (Fig. 157), B being charged positively and A negatively. The electric field is shown in (i). If now the gap AB becomes conducting, say by the occurrence of a spark, the charge on B moves upwards and that on A downwards and the lines of force move inwards. The travel of the lines inwards implies the magnetic field (p. 175) and when the discharge is just complete, the lines are moving inwards (ii) and the magnetic field is greatest. This magnetic field causes the current to continue (p. 177) until the charges are reversed (iii) and the electric field is as shown. Only half the field is drawn in the diagram, for the sake of simplicity. The current will now start in the reverse direction, and will continue to oscillate backwards and forwards until the energy is all dissipated.

In Fig. 158 the more distant lines LMN are shown as well as the near ones EFG and ABC. The behaviour of the smaller lines has been described. The more distant ones will not reach the gap by the time the reversal of charge is

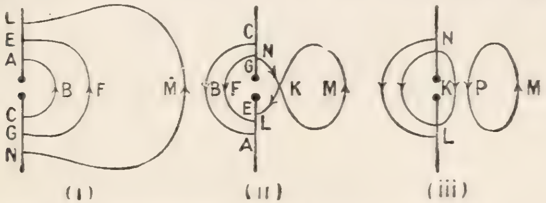


FIG. 158.—Electric lines of force forming radiation loops.

complete, and they form loops such as KM (ii). The loop is unstable and breaks into a short line LKN and a separate loop PM (iii). When the next half-oscillation occurs, the

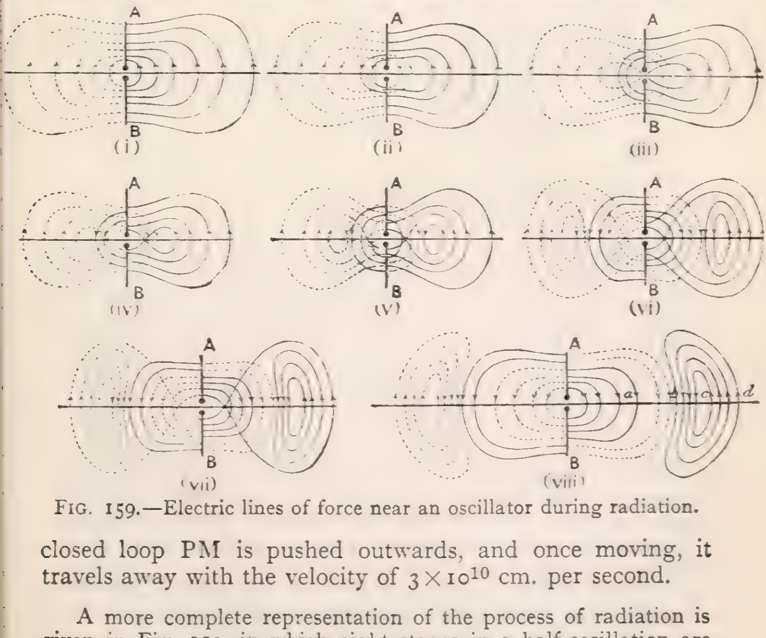


FIG. 159.—Electric lines of force near an oscillator during radiation.

closed loop PM is pushed outwards, and once moving, it travels away with the velocity of 3×10^{10} cm. per second.

A more complete representation of the process of radiation is given in Fig. 159, in which eight stages in a half-oscillation are

shown. The lines originally on one side of the oscillator AB are shown in full line and those on the other in dotted line.

Radiation from an aerial.—In practice it is now customary to suppress the lower half of the Hertzian oscillator by connecting to earth, while the upper half is a tall wire or mast known as an **aerial.** Remembering also that the upper layers of the atmosphere FG (Fig. 160) are electrically conducting, as is the earth, the waves radiated form large cylindrical surfaces with the aerial as axis; it will be seen that at the instant shown, the electric field at *a*, *c* and *e* is directed downwards, and at *b*, *d* and *f* upwards. The accompanying magnetic field is, of course, horizontal and

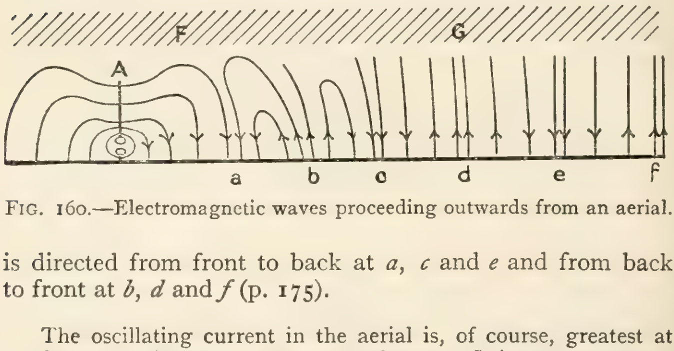


FIG. 160.—Electromagnetic waves proceeding outwards from an aerial.

is directed from front to back at *a*, *c* and *e* and from back to front at *b*, *d* and *f* (p. 175).

The oscillating current in the aerial is, of course, greatest at the bottom and decreases to zero at the top. It is now customary to provide a horizontal wire or conductor at the top. This is equivalent to putting a capacity there, and the current flows up and down the aerial between the cross-piece and earth. Thus the current at the top is no longer zero and the radiation is therefore as great as would be effected by a much taller aerial without the capacity at the top.

Wave-length.—It is a frequent practice to express the constants of electromagnetic waves in terms of wave-length instead of frequency. If n waves are emitted by the oscillator in one second, the distance reached by the first wave at the end of this second is equal to the velocity V of the waves. Also if λ is the length of one wave, or the distance between any point in space and the next point in which the phase of

the wave is exactly the same as at the first point, the length covered by n waves is $n\lambda$.

$$\therefore \ n\lambda = V$$

In the case of electromagnetic waves in space unoccupied by matter, the velocity V is 3×10^{10} cm. per second, and for air the velocity is very nearly the same.

$$\therefore \ n\lambda = 3 \times 10^{10} \text{ cm. per second}$$

If the wave-length in a given case is, say, 500 metres, or 5×10^4 cm.—

$$n \times 5 \times 10^4 = 3 \times 10^{10}$$
$$\therefore \ n = 6 \times 10^5$$
$$= 600{,}000 \text{ oscillations per second}$$

Reception of electromagnetic waves.—If the waves from an aerial fall upon a conductor AB (Fig. 161) with a minute

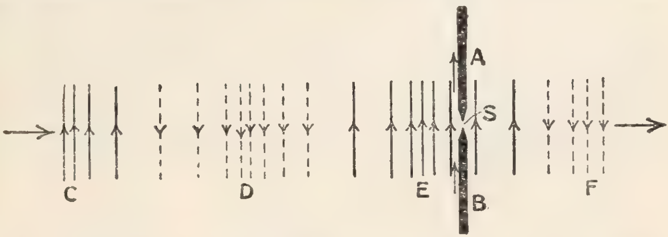


FIG. 161.—Electromagnectic waves producing oscillation in a detector

gap at its central part S, then when the arriving electric field is directed upwards as at C and E, there will be an e.m.f. in B driving a current upwards. When the parts D and F of the wave arrive there will be a current in AB directed downwards. If these e.m.f.'s are sufficiently great, the current will flow across the gap and a minute spark will be seen there. This is the original mode of detection of the waves employed by Hertz. The effect may be very much increased if the natural time of oscillation of the current up and down AB is the same as the periodic time of the incident waves, for then each wave will arrive at the proper time to increase the oscillatory current due to the preceding wave. This effect is called **syntony** or **tuning,** and is of general application in wireless telegraphy and telephony for the detection of waves which would otherwise be too feeble to produce any effect.

Detectors.—It is not possible to give here a complete history or account of wireless telegraphy and telephony, but it may be noted that the Hertz detector gave place to a much more sensitive detector, namely, the **coherer,** consisting of a quantity of metal filings in the gap. The minute sparks caused them to cohere and the electrical resistance of the circuit dropped so that a local battery could produce a large current and ring a bell or work a sounder. Then came the magnetic detector and later the crystal detector, which is still largely employed. The oscillatory current at the receiving station, produced by the incoming waves, passes through a telephone receiver T (Fig. 162); but the frequency of the current is from 24,000 to 1,000,000 per second. This is far too rapid to cause vibration of the diaphragm of the telephone, and even if it could, the ear can only detect vibrations of frequency of about 22,000 per second, and this is a very shrill note A crystal C, of some

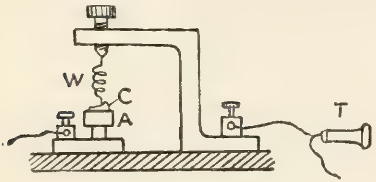


FIG. 162.—Crystal detector.

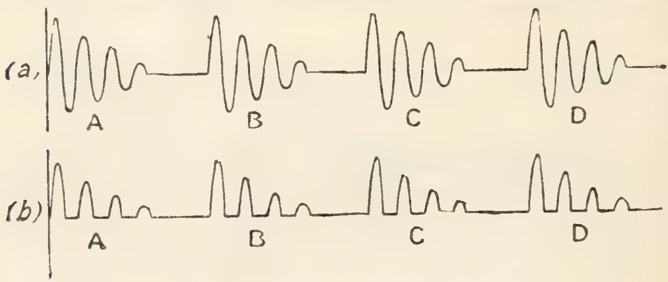


FIG. 163.—"Rectification" of oscillations.

such substance as silicon or herzite is therefore mounted in fusible solder in a metallic cup A, and a wire W lightly touches the crystal. The contact of wire and crystal has a much higher resistance for current flowing in one direction across the contact than for current in the opposite direction, so that half the oscillatory current is very nearly suppressed. If therefore A, B, C and D (Fig. 163 (*a*)) represent currents

due to arriving trains of waves, the crystal detector suppresses half of each wave, and the resulting currents in the telephone are as given in Fig. 163 (*b*). Thus A, B, C and D will each produce an impulse on the diaphragm of the receiver, and the ear will detect a sound corresponding to the frequency of these impulses.

A common type of circuit for receiving wireless signals by means of the crystal detector is shown in Fig. 164. The aerial circuit may be tuned by the variable inductance L_1, which is the primary of a transformer of which L_2 is the secondary. The secondary circuit may be tuned by the variable condenser C_1. C is the crystal and T the telephone receiver.

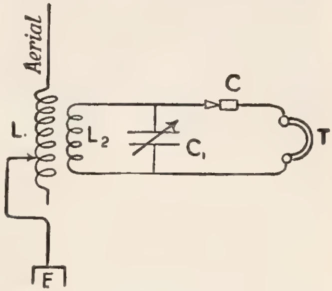


FIG. 164.—Receiving circuit with crystal detector.

Valves.—The practice of wireless telegraphy and telephony has been revolutionised by the discovery of the **triode**, or valve with three terminals. It consists of a glass bulb of

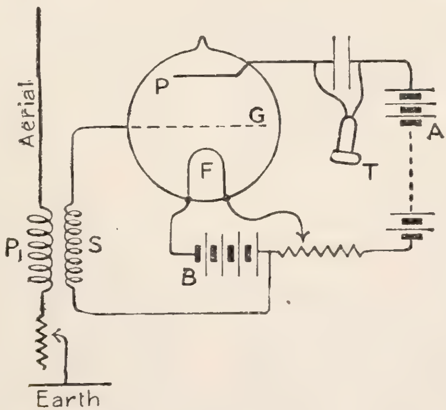


FIG. 165.—Receiving by means of the triode.

very high vacuum, with a hot filament F (Fig. 165) maintained in incandescence by a battery B. It will be seen later

that a hot filament emits swarms of electrons (p. 190) which are particles of **negative** electricity. G is a perforated grid or conducting network, and when G is at a lower potential than F the electrons are driven back upon F; but if G is at a higher potential than F the electrons pass freely towards G. The potential of G therefore controls the supply of electrons passing from F, and since the electrons are the only carriers of the current in the vacuum, the potential of G controls the current flowing on to, or away from, F. Now the high voltage battery is connected to the third electrode or plate P, and the current it can produce is controlled by the potential of G. Thus very small oscillations in potential of G, produced by the aerial current in P, and secondary coil S, cause comparatively large variations in current in the circuit FAP. The valve therefore acts as an **amplifier,** and by applying the plate variations in potential to a grid of a second valve, further amplification can be obtained. Using a series of several valves in this way the incoming oscillations may be amplified thousands of times.

Valve as detector.—The triode may also be used as a detector in place of the crystal detector (p. 182), for under suitable conditions the oscillations in potential of the grid on either side of a particular value do not produce equal variations in current in the plate circuit, so that the oscillations are **rectified.** This, however, involves additional resistance and capacity to the circuit, which are not shown in Fig. 165.

If the e.m.f. between grid and filament be measured and at the same time the current in the plate circuit, a curve such as shown in Fig. 166 may be obtained. If, then, O*e* is the mean potential of the grid, and the oscillations *eg* and *ef* about this mean are produced by the waves received, *ab* is the change of current obtained in one direction in the plate circuit, and *ac* that in the other. Thus there will be a balance of current in one direction in the telephones over that in the other. This is the

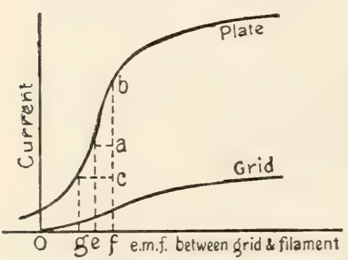


FIG. 166.—Diagram showing the amplifying and rectifying effect of the triode.

condition for hearing the signals. The rectification is, however, not nearly so complete as in the case of the crystal detector, but the amplification produced by the valve more than compensates for this.

Production of waves.—In the early days of wireless telegraphy, the only method of producing the electromagnetic oscillations was that of using an induction coil for the production of a spark in the oscillating circuit (p. 146).

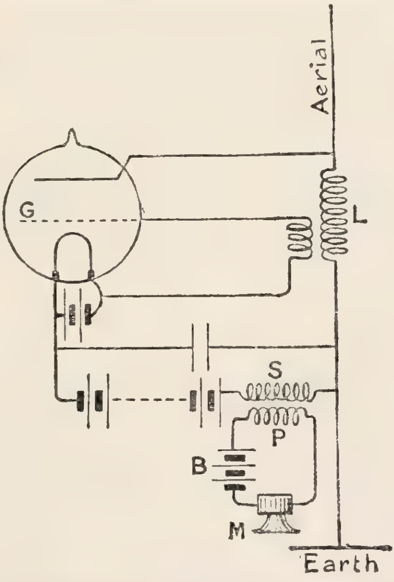


FIG. 167.—Employment of the triode in wireless telephony

At first, batteries supplied the primary current for the induction coil, but later, alternators and transformers were employed. For the production of the continuous waves required for wireless telephony, although other methods are used, the valve has supplied an extremely efficient source. For, if the plate and grid circuits are coupled by a transformer L (Fig. 167), connected suitably, any variation in current in the aerial circuit produces oscillations in the grid circuit, which are in such a direction that they react on the plate

circuit and increase its oscillations. Thus when the proper condition is reached, violent oscillations are produced in the aerial, the source of energy being the high voltage battery in the plate circuit. In order to modify these oscillations a telephone microphone M and transformer PS impress the variations caused by the speech upon the aerial circuit. If the variations in current caused in S are represented by curve (*a*) (Fig. 168), the oscillations (*b*) are modified in

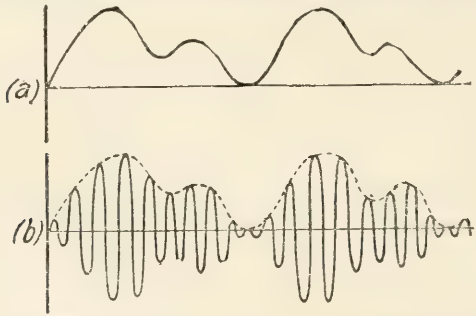


Fig. 168.—Modification of the electromagnetic waves for wireless telephony.

intensity in a similar manner, and these modifications are heard in the telephones in the receiving station.

Exercises on Chapter XII

1. Explain how magnetic field may be considered as the motion of an electric field.

2. Describe how the oscillatory nature of the discharge of a condenser may be investigated experimentally.

3. Calculate the number of oscillations per second of the current in a circuit of capacity 0·05 microfarad and inductance 2 millihenries.

4. Find the wave-length of the radiation from a circuit of capacity 0·11 microfarad and inductance 4 millihenries.

5. Give an account of some method of detecting electromagnetic waves.

6. Explain the necessity of rectification of electromagnetic oscillations and the method of carrying it out by means of a crystal detector.

7. Describe how a triode valve may act as an amplifier.

8. Describe how a valve may be used as a generator of oscillations.

9. Calculate the vibration frequency of the oscillations when electromagnetic waves of length 369 metres are emitted.

10. Explain how electromagnetic waves may arise when a condenser is discharged.

CHAPTER XIII

CURRENT IN GASES—X-RAYS

Gases at low pressure.—The electric current will not pass through air at the atmospheric pressure unless very high electromotive forces are employed, and then the current generally takes the form of a spark. If, however, the air in a glass tube is pumped out until the pressure in the tube is about a quarter of an atmosphere, the current passes much more easily, and at a hundredth of an atmosphere the current passes very easily and the path of it is luminous,

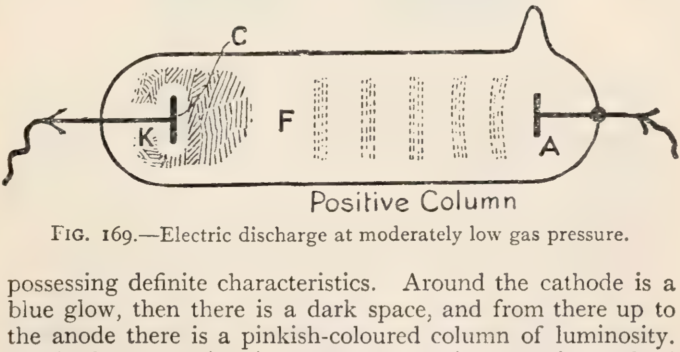


FIG. 169.—Electric discharge at moderately low gas pressure.

possessing definite characteristics. Around the cathode is a blue glow, then there is a dark space, and from there up to the anode there is a pinkish-coloured column of luminosity. On further reduction in pressure the discharge is resolved into parts illustrated in Fig. 169. The coloured glow, called the positive column, proceeds from the anode A, and consists of disc-shaped **striations** separated by dark spaces, and at the end of the positive column, separating it from the cathode glow, is the **Faraday dark space** F. As exhaustion proceeds, a second dark space C, between the cathode and the cathode glow, becomes apparent. This is called the **Crookes dark space.** With continuing reduction in pressure, the scale of

the whole phenomenon grows, and it grows from the cathode. First the positive column disappears as there is no longer room for it in the tube; then the Faraday dark space, and then the cathode glow, until at the highest stage of exhaustion attainable the Crookes dark space fills the whole tube.

Cathode rays.—When the Crookes dark space fills the tube, the walls become luminescent with a greenish glow characteristic of the kind of glass from which the tube is made. A metallic obstacle in the tube casts a shadow upon the walls (Fig. 170) showing that something is travelling in

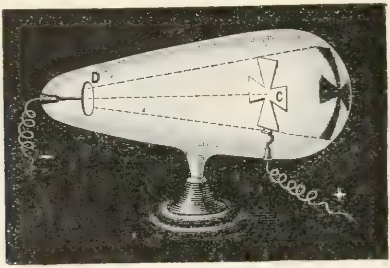


FIG. 170.—Luminescence produced by cathode rays.

straight lines from the cathode and causing the luminescence of the glass walls of the tube. Also light movable bodies are driven away from the cathode. The radiation from the cathode is called the **Cathode rays,** and if they are caught in a hollow conductor they are found to carry **negative charges of electricity.**

One of the most important properties of the cathode rays is the deflection which they experience in traversing a magnetic field. If the beam of rays be cut down by means of a screen, and a magnet be brought near this beam, it is found to be deflected exactly as an electric current would be. That is, the deflection is at right angles both to the path of the rays and to the magnetic field. But on applying the law given on p. 133 it is found that the current is a movement of **negative electricity** away from the cathode. This property enabled Sir J. J. Thomson to discover the nature of the cathode rays.

The beam is limited by two metallic screens A and B (Fig. 171) and in the absence of magnetic field falls on the end of the tube at P. A screen of zinc sulphide is placed here and the incidence of the rays upon it causes a bright patch of blue luminescence.

If now a magnetic field restricted to the space MM is applied, the patch of luminescence moves to Q and by measuring PQ, and from the geometry of the figure, it may be found that the rays in the region of the magnetic field take a circular path the radius of which we will take as r cm. If the rays consist of particles of

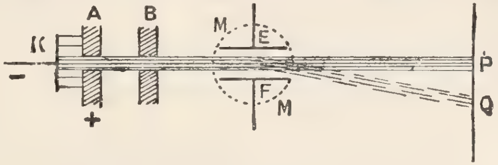


FIG. 171.—Deflection of cathode rays.

mass m grm. having a charge of electricity e electromagnetic units and moving with velocity v cm. per second, each particle is equivalent to an electric current ev and the force on it at right angles to its path is Hev (p. 133). If it moves in a circle of radius r, its centrifugal force is mv^2/r; equating these two we have—

$$\frac{mv^2}{r} = Hev$$

or,
$$\frac{m}{e} . v = Hr$$

The magnetic field is now replaced by an electric field E between the plates E and F, and if the particles have negative charge e, the force on each is Ee. The electric field therefore produces a displacement of the spot at P, but by applying the electric and magnetic fields together it may be so arranged that the forces due to them are equal and opposite and the spot remains at P.

Then,
$$Ee = Hev$$

or,
$$v = \frac{E}{H}$$

Since the fields E and H are known, the velocity v of the particles is found; then from the experiment with the magnetic field alone $\frac{m}{e}$ is determined.

It was found by Sir J. J. Thomson that v varied, although its value was generally between 2×10^9 and 3×10^9 cm. per second. But the value of m/e is the same, whatever the gas in the tube or whatever the metal of which the electrodes are made. The value of m/e from later and more accurate determinations is $5{\cdot}64 \times 10^{-8}$

absolute electromagnetic units, and it is clear that this quantity is of the nature of an electrochemical equivalent, for it is a mass per unit charge of electricity. Now the electrochemical equivalent of the hydrogen ions in electrolysis is $1\cdot044 \times 10^{-4}$ in absolute electromagnetic units. It follows that the electrochemical equivalent of the corpuscles in the cathode rays is about $\frac{1}{2000}$ of that of the hydrogen ions in electrolysis. It was found eventually that the amount of electricity carried by the ion is in both cases the same, and that the mass of the corpuscle in the cathode rays is $\frac{1}{1850}$ of the mass of the hydrogen atom.

Electrons.—The negative corpuscles first found in the cathode rays are now known to be of universal occurrence. They are constituents of every material atom and are liberated at high temperatures from all matter, and by the action of ultra-violet light from many substances. The name **electron** has been given to them, and they are the smallest part of matter known, being the ultimate smallest quantity of electricity which exists. The actual charge of the electron measured in ordinary units is $4\ 77 \times 10^{-10}$ electrostatic unit or $1\cdot59 \times 10^{-20}$ absolute electromagnetic unit.

Mass and charge of electrons.—The experiment described on p. 190, and many kindred experiments, gave a value for m/e but did not give the separate values for m and e. The first experiment to give the separate values of m and e was the cloud experiment of C. T. R. Wilson, in which the electrons were liberated in air in which the water vapour present was super-saturated. In this case condensation takes place on the electrons and a cloud is formed From the rate at which the cloud settles, the size of the drops can be found, and eventually the number of drops per unit volume and the charge associated with each drop. The method was, however, improved by Millikan, who separated individual drops for examination.

It is known that a small spherical drop in air falls with the constant velocity of $\frac{2}{9} \cdot \frac{ga^2}{\eta}$, where g is the acceleration of gravity (981 cm. per second per second), η is the coefficient of viscosity of the air, which is known, and a is the radius of the drop. In Millikan's experiment oil or water in fine spray traverses a space containing electrons and a drop may acquire one or more electrons. The drops pass through a space in which there is a vertical electric field E (p. 38) in such a direction that the negative charge of the

electron experiences a force Ee driving the drop upwards. If Ee=mg the drop is just suspended, if E$e>mg$ the drop rises, and if E$e<mg$ the drop falls. In this way all those drops are removed but those for which E$e=mg$.

Having separated out such a drop, the electric field is removed and the drop is allowed to fall under gravity and its rate of fall v is measured.

Now, $$v=\frac{2}{9}\cdot\frac{ga^2}{\eta}$$

and if the drop is water, $$m=\frac{4}{3}\pi a^3, \text{ or, } a=\left(\frac{3m}{4\pi}\right)$$

$$\therefore\ v=\frac{2}{9}\cdot\frac{g}{\eta}\left(\frac{3m}{4\pi}\right)^{\frac{2}{3}}$$

or, $$\frac{3m}{4\pi}=\left(\frac{9}{2}\cdot\frac{\eta}{g}\right)^{\frac{3}{2}}v^{\frac{3}{2}}$$

but, $$mg=\mathrm{E}e$$

$$\therefore\ \frac{3}{4\pi g}=\left(\frac{9}{2}\cdot\frac{\eta}{g}\right)^{\frac{3}{2}}v^{\frac{3}{2}}\frac{1}{\mathrm{E}e}$$

or, $$e=\frac{4\pi g}{3\mathrm{E}}\left(\frac{9}{2}\cdot\frac{\eta}{g}\right)^{\frac{3}{2}}v^{\frac{3}{2}}$$

Thus the value of the electronic charge is found. Some drops had two electrons, others three, but the electronic charge itself was clearly evaluated.

Röntgen or X-rays.—Probably the most important property of the cathode rays is the production of a further kind of radiation when they fall upon matter, particularly matter of considerable density. These new rays were discovered by Röntgen and are sometimes called **Röntgen rays,** but more commonly **X-rays.** X-rays have great penetrability, they produce luminescence in various minerals, they affect the photographic plate, and, most important of all, they render the gas through which they pass, a conductor of electricity.

The early form of X-ray tube contains a concave cathode K (Fig. 172), so that the cathode rays, which travel normally from the cathode, are focussed upon a sheet of platinum A, called the anti-cathode. This may be used as the anode for the tube, or in many cases a separate anode is provided. The spot upon which the cathode rays impinge becomes hot,

sometimes red hot, and is the source of the X-rays. This fact may be proved by placing a lead screen C with holes *e*, *g*, etc., in it, near the X-ray tube, and beyond this a photographic plate D. On developing the plate, exposed patches are found at *f*, *h*, etc., and on replacing the plate and drawing the lines *fe*, *hg*, etc., these lines will be found to meet at a point upon A, which is therefore the origin of the X-rays.

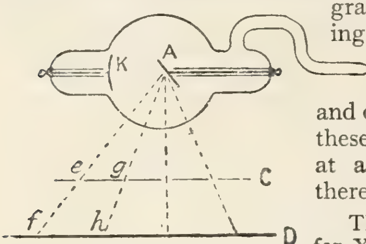


FIG. 172.—Origin of X-rays.

The variation of penetrability for X-rays with the density of the material gives rise to the **radiographs** of bones and other organic members, used in surgery and so widely known.

A more modern form of X-ray tube, the **Coolidge tube,** is shown in Fig. 173 (Plate II). The air is removed as completely as possible from the tube and the supply of electrons for the discharge is produced by using a spiral of tungsten wire inside a cup-shaped cathode. The tungsten wire is heated by an electric current, and at the high temperature emits a copious supply of electrons. By means of an induction coil or a transformer, the electric field is produced which drives these electrons against a massive tungsten block. In some cases the block is water-cooled. The tube illustrated is surrounded by a heavy lead-glass shield which cuts off most of the X-rays, except where a window is situated for the passage of the rays for experimental purposes. The X-rays produce bright luminescence in barium platino-cyanide, and a screen of this material is used instead of the photographic plate when the radiograph is to be observed directly.

Ionisation.—To the property of the X-rays of rendering the gas through which they pass conducting, the name of **ionisation** has been applied ; the gas is said to be **ionised.**

If an electroscope (p. 40) be charged, and an X-ray tube send a beam of X-rays into the chamber of the electroscope, the leaves rapidly collapse because the air surrounding them becomes conducting. The explanation of this conductivity is that the atoms of the gas are caused by the X-rays to loose an electron. Thus the electron is a free negative charge of electricity and the remainder of the atom is now positively charged, owing to the loss of the electron. That the atom was originally neutral is proved by the fact that

PLATE I.

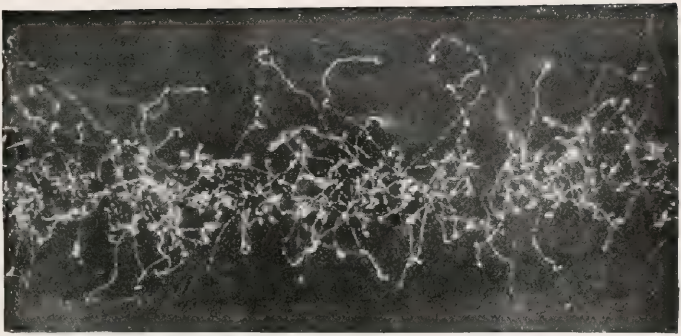


FIG. 174.—(*a*) Photograph by C. T. R. Wilson of the path of a beam of X-rays through air supersaturated with water vapour, showing the cathode or β-ray tracks produced. Magnification $2\frac{1}{2}$ diameters.

[*From the "Proceedings" of the Royal Society.*]

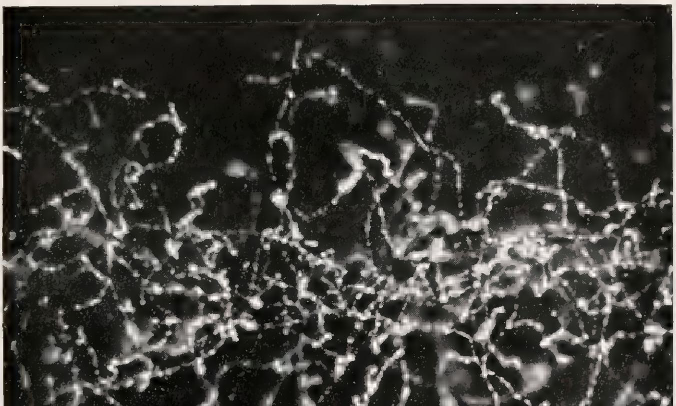


FIG. 174.—(*b*) Photograph by C. T. R. Wilson of the path of a beam of X-rays in air supersaturated with moisture. Magnification 6 diameters.

[*From Kaye's "X-Rays."*]

[*To face p.* 192]

PLATE II.

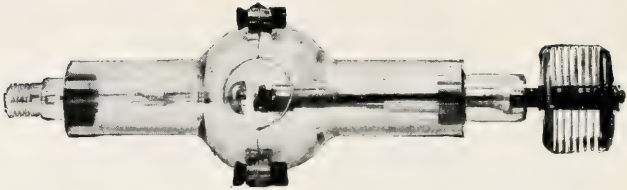


FIG. 173.—Coolidge X-Ray tube in lead glass shield.

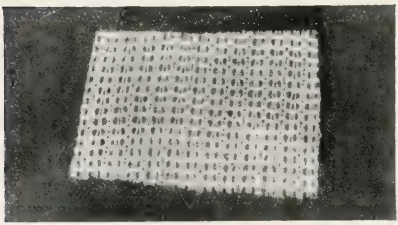


FIG. 179.—Photograph taken by placing a piece of incandescent gas mantle in contact with sensitive plate.

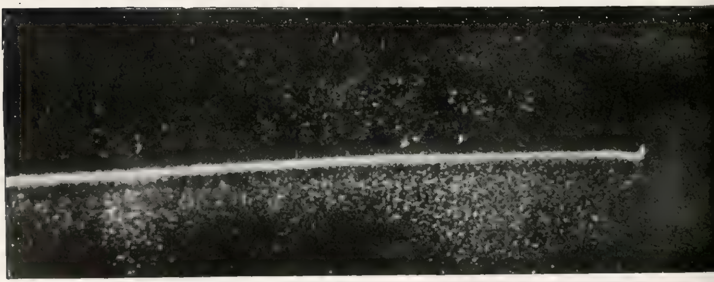


FIG. 181.—Photograph by C. T. R. Wilson of the track of an α particle from radium through air supersaturated with water vapour.

[*From the "Proceedings" of the Royal Society.*

charged conductors in air do not loose their charge by conduction through the air, although their surfaces are being bombarded by millions of atoms per second. Both electrons and the positive remainders of the atoms experience force in the electric field. If a conductor is positively charged, the electrons are attracted towards it and its positive charge soon becomes neutralised. If the conductor is negatively charged, the positive charges, or **ions,** are attracted towards it and again its charge is neutralised. The electrons and the positively charged atoms soon become loaded with neutral atoms so that in the electric field they move with only small velocities through the gas. In this respect they resemble very much the ions in electrolysis (p. 120).

The electrons when liberated from the atoms of the gas travel at first with high velocity and other atoms of the gas are **ionised by collision.** It has already been mentioned that in a gas supersaturated with water vapour, condensation upon ions occurs, and this has been made use of by C. T. R. Wilson to exhibit in a very beautiful manner the effect of a beam of X-rays passing through the gas. Fig. 174 (Plate I), is taken from a photograph of the gas just as a beam of X-rays passes through it, and the tracks of the electrons are marked by little clouds of minute drops, produced wherever the electron ionises an atom of the gas. The tracks cease where the velocity of the electron has become so reduced that it no longer causes ionisation by collision.

Ionisation current.—In order to examine the conductivity of the ionised gas, the arrangement shown in Fig. 175 may

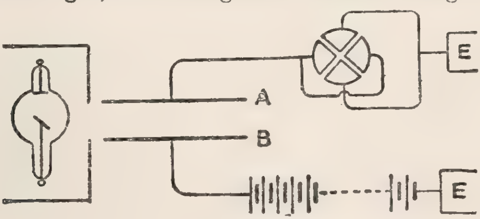


FIG. 175.—Measurement of ionisation current.

be employed. The X-rays from a tube pass between two parallel plates A and B. A is connected to an electrometer and B to the pole of a battery of many cells. No current passes unless the air between A and B is ionised. But on producing the X-rays, the current passes and A becomes

charged. The rise (or fall) of potential of A is observed by the changing deflection of the electrometer, and if the capacity of A and the electrometer together is known, the amount of charge received per second or the current through the gas is known. The p.d. between the plates is known from the battery employed. If then the current for varying p.d. is found and plotted in a graph (Fig. 176), it is seen that with increasing p.d. the current increases rapidly, but its rate of increase gets less until eventually an increase in p.d. does not produce any increase in current. The current at this stage is called the **saturation** current, and in this condition all the ions produced by the X-rays are used up in carrying the current. Eventually, if the p d. is continuously increased, the curve bends up sharply. This occurs when the velocity of the ions moving towards the plates A and B becomes sufficient to ionise the gas, thus producing a copious supply of ions for carrying the current.

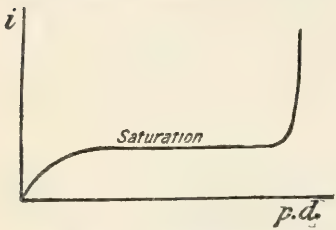


FIG. 176.—Ionisation current.

Positive rays.—In the discharge tube the cathode rays or streams of electrons travel away from the cathode. But if there are any positively charged bodies moving in the opposite direction they would not be observed unless they produce striking effects. Such rays do exist but have much smaller velocities than the electrons, and their presence was only detected when a perforated cathode was used, so that these positively charged bodies passed through the cathode. They formed faintly luminescent streamers behind the cathode and from the mode of their production they were first called **canal rays**, although the name **positive rays** is generally employed. An application of the method applied to the cathode rays (p. 189) soon showed that the velocity is less in the case of the positive rays, and that the mass and electric charge carried was not a universal constant as in the case of electrons, but varies from material to material and even with the same material.

Positive ray analysis.—Sir J. J. Thomson has modified the method applied to the cathode rays to enable the properties of

the positive rays to be studied. The cathode A (Fig. 177) is pierced by a very fine hole running axially, so that positive ray particles acquiring a velocity in the discharge chamber B, and travelling in the proper direction, pass down this hole and produce a thin beam of positive rays which falls on the photographic plate

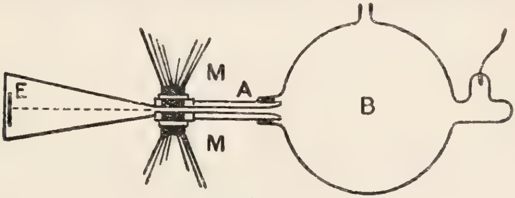


FIG. 177.—Positive ray analysis.

or luminescent screen E. On their way the rays pass between the poles MM of a strong magnet, and the magnetic field would cause the rays to be deflected horizontally (p. 188). MM are also insulated and are connected to the opposite poles of a battery, so that the electric field between them causes the positive rays to be deflected vertically (p. 190). The horizontal deflection caused by the magnetic field is proportional to $\frac{e}{m}\cdot\frac{1}{v}(=x)$, and the vertical deflection produced by the electric field is proportional to $\frac{e}{m}\cdot\frac{1}{v^2}(=y)$.

$$\therefore \frac{x}{y}=k.v$$

and all points lying on a straight line passing through O (Fig. 178) correspond to particles having the same velocity. Also, $\frac{x^2}{y} = \mathrm{K}\cdot\frac{e}{m}$, so that all particles having the same value of e/m fall along a parabola such as BA or DE. Taking the same value of y, such as OF, it follows that the ratio of HF^2 to GF^2 is equal to the ratio of e/m for one particle to e/m for the other.

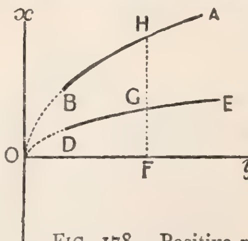


FIG. 178.—Positive ray figures.

By using different gases in the bulb B (Fig. 177), it has been found by examining the parabolas obtained, that the simplest positive ray particle is the hydrogen atom which has lost one electron, but nearly all the other elements have been identified, and many molecules. For example, the oxygen atom may lose either one or two electrons and the molecule O_2 one electron, and O_3 one electron, while the mercury atom may lose as many as eight electrons.

Aston has modified the Thomson apparatus so that the electric field splits up the fine beam of positive rays, and the magnetic field recombines it in such a way that the atoms having different values of e/m are brought to foci at different lines, giving a spectrum which has been called the **mass spectrum.** In this way he has shown that substances such as chlorine having fractional atomic weights (35·46) are really mixtures of atoms having whole-number atomic weights, which in the case of chlorine are 35 and 37. The two sets of atoms have identical chemical properties and are called **isotopes.**

Nature of X-rays.—It was at first thought that X-rays might be particles of matter travelling with very high velocity, but this idea was soon dismissed on account of their extremely great penetrating power and the fact that they are not, like cathode rays, affected by an electric or a magnetic field. Then it was considered that they are electric pulses started by the stoppage of the electron. But it was found that X-rays falling on matter produce secondary X-rays which are characteristic of the material. All attempts to produce interference and diffraction, such as can be produced with light waves, failed for a time, until crystals were used to replace the diffraction gratings used for light waves. It was then found that the atomic structure of the crystal was fine enough to produce diffraction, and although the diffraction effects are extremely complex it has been found possible not only to determine from them the arrangements of the atoms in many crystals, but to establish the fact that X-rays are of the same nature as light but of much shorter wave-length. The smallest wave-length of the X-rays is about 10^{-8}, or 0·00000001 cm., while ordinary light has a wave-length of about 5×10^{-5} or 0·00005 cm.

Exercises on Chapter XIII

1. Describe the general appearance of the electric discharge through gases as the pressure of the gas is reduced.
2. What are cathode rays? Describe their chief properties.
3. How has the nature of the cathode rays been investigated?
4. How have the mass and charge of an electron been found?
5. Describe some form of X-ray tube.
6. State what you know about X-rays.
7. How may the ionisation of a gas be measured?
8. What are positive rays, and how has their nature been investigated?

CHAPTER XIV

RADIOACTIVITY

Discovery.—It has long been known that certain substances exposed to light absorb energy which they afterwards give out as light. This property was being investigated by Becquerel in 1896, when it was found that a certain salt of uranium emitted a radiation which could affect the photographic plate although the salt had not been exposed to light. This startling discovery led to a search for other bodies which could produce the same effect, and it was found that thorium had the same property as uranium. Thorium is a substance used in the manufacture of incandescent gas mantles. If such a mantle be placed in contact with a photographic plate in the dark for about a fortnight, then on developing the plate, the structure of the mantle will be found delineated upon it, the nearer parts of the mantle having acted upon the plate to a greater extent than the more distant parts. Fig. 179 (Plate II) is from a photograph obtained in this way.

Ionisation.—Even more important than the photographic effect is the fact that the radiations from uranium and thorium ionise the air in the space surrounding these bodies.

If a small quantity of a uranium or a thorium salt be placed in the chamber of a charged electroscope (p. 40), it will be found that the leaves slowly collapse. An extremely efficient electroscope for examining such radiations has been devised by C. T. R. Wilson and is illustrated in Fig. 180. The leaf A is of aluminium foil and depends from the wire B held by a conductor C in the insulating tube D. By depressing the terminal J a light conductor G is brought into contact with C and the leaf can be charged. On releasing J, the spring H raises G and the leaf is insulated. A microscope with scale in the eyepiece is generally used for observing the deflection through the window A, and the travel across the

scale measures the rate of leak of the charge. The uranium or thorium is spread on a sheet under the trap K, which is closed by thin tissue paper through which the radiations pass into the chamber of the electroscope.

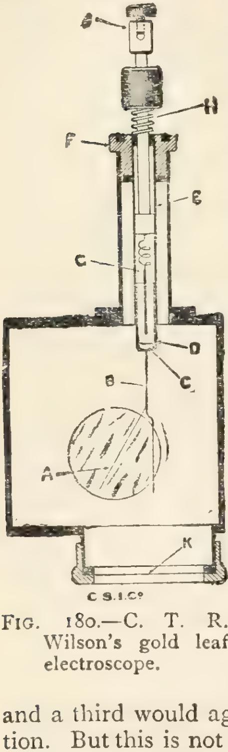


FIG. 180.—C. T. R. Wilson's gold leaf electroscope.

α, β and γ rays.—It was due to Sir Ernest Rutherford that the complex nature of the rays emitted by radium and thorium became understood. Their nature was examined by measuring their penetrating power. If a layer of thorium oxide be placed beneath the trap K of the electroscope (Fig. 180), then with the leaf charged, the time for it to make a certain travel across the field of the microscope can be measured. The intensity of the ionisation produced in the chamber of the electroscope by the rays from the thorium is then inversely proportional to this time of collapse. On placing a layer of tinfoil or aluminium foil over the thorium oxide the ionisation in the chamber is reduced in a certain ratio, say by 10 per cent. If the rays are all of one kind, a second layer of foil would produce the same percentage reduction in the rays as did the first, and a third would again produce the same percentage reduction. But this is not found to be the case. The second sheet produces hardly any reduction in the rays transmitted. Hence the rays are complex, the first kind being largely absorbed by the first screen, and the remainder being absorbed very slightly. These most absorbable rays are called α **rays.**

If the α rays are removed by a single screen and those remaining are examined by lead screens about 2 mm. in thickness, a similar phenomenon is observed, the first lead screen producing a large reduction in the intensity of the rays and further lead screens producing very small reductions. The rays removed by the first lead screen are called β **rays** and the remainder γ **rays**. The following table, due to Sir

Ernest Rutherford, gives the relation in penetrating powers in the radiations very simply :—

Rays.	Thickness of aluminium which reduces ionisation to one-half.	Relative penetrating power.
α	0·0005 cm.	1
β	0·05 cm.	100
γ	8·0 cm.	10000

Radioactivity.—The power possessed by some substances of emitting α, β and γ rays is called **radioactivity.** It is known to be possessed by uranium, thorium, radium and actinium. A search amongst the rare metals resulted in **radium** being discovered by Mme. Curie in pitchblende, a mineral rich in uranium. Radium was discovered by separating continually the more radioactive part from the pitchblende by chemical means and ultimately by fractional crystallisation. Radium occurs in very minute quantities, but is vastly more radioactive than uranium or thorium.

α rays.—The α rays have small penetrating power, but produce luminescence in various minerals such as diamond and zinc sulphide, either of which may, in the dark, be seen to glow if brought near a minute quantity of radium. If a screen be made by spreading a layer of zinc sulphide upon paper, the screen will, on being examined by a low-power microscope in the dark in the presence of radium, be luminous. The luminescence will have the appearance of a shower of sparks. Each α ray particle as it strikes the screen causes a flash. By using a high magnification and counting the flashes in a given area of screen per minute the number of α particles emitted per second has been found.

α rays can with difficulty be deflected by a magnetic field, but the deflection is in the **opposite direction** to that in the case of the cathode rays. If the α rays be examined in the electric and magnetic fields as in the case of the cathode rays, p. 189, it will be found that they have **positive charges.** It is now known that the α ray particle has an atomic weight 4, and carries a charge equal to twice that of the hydrogen atom in electrolysis. In fact the α-ray particles are **helium atoms** which have lost two electrons.

α rays possess great ionising power, and in passing through air leave an ionised track behind them. This is well seen in the

photograph by C. T. R. Wilson, Fig. 181 (Plate II), made by the passing of an α-ray particle through air supersaturated with moisture. The considerable mass of the particle makes the track much more nearly straight than in the case of the electron (compare Fig. 174), the mass of the α-ray particle being 7400 times as great as that of the electron. It is interesting to note that the path ends abruptly. This means that as the velocity falls, the particle ceases suddenly, at some particular velocity, to be able to produce ionisation of the air. This critical velocity is about 10^9 cm. per second. The length of path of the α ray in air before ionisation ceases, is an important quantity known as the **range** of the α ray and is characteristic of the material from which it arises. The ranges of the α rays from various substances may be seen in the table on p. 203. The range depends upon the initial velocity, or velocity of emission of the α particle, and from this it is possible to find the life of the substance which emits it (p. 202).

β **rays.**—Examination of the β rays proves them to be electrons, moving with high velocity, which in some cases attains the value $2{\cdot}85 \times 10^{10}$ cm. per second. This is the highest actual known velocity of any material substance, and approaches very nearly the velocity of light 3×10^{10} cm. per second. The β rays, like cathode rays, are readily deflected by a magnetic field. It is to the β rays that the greater part of the photographic action of the radiation from radioactive substances is due.

γ **rays.**—A magnetic field has no effect on the γ rays, which are the same in character as X-rays. γ rays have, however, in most cases a greater penetrating power than X-rays, which shows that they are of shorter wave-length.

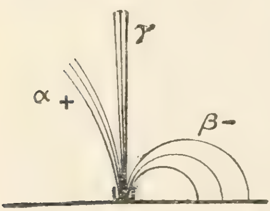


FIG. 182.—Diagram of α, β and γ rays.

A simple diagram due to Mme. Curie illustrates very well the distinctive characteristics of the three kinds of ray emitted by radioactive substances. If the capsule in Fig. 182 contains a small quantity of radioactive material and a magnetic field be supposed to be present, whose direction is from front to back, the α rays are slightly deflected to the left, the β rays more deflected to the right, while the γ rays are undeflected.

Origin of the rays.—From the large amount of energy with which the rays are emitted, it is clear that there is a considerable store of energy in the atom. It has been found that 1 grm. of radium emits 135 calories per hour, and during its whole life (p. 205) it probably emits about 10^{10} calories. The formation of 1 grm. of water by the combustion of hydrogen in oxygen liberates 3,900 calories. This comparison shows what immense stores of energy there must be in atoms, and with corroboration by the kind of light that can be emitted by atoms, shows that some internal part of the atom is rotating with great velocity. Now all the radioactive materials have very complex atoms, as is shown by their high atomic weight. These complex atoms are not very stable, and every now and then one undergoes a change in structure with the throwing off at high velocity of an α particle, or charged helium atom. The high momentum of the α particle involves an equal and opposite momentum of the atom from which it comes, but on account of its mass being much greater than that of the α particle its velocity is much less. Such atoms are called **recoil atoms.**

The liberation of an α particle diminishes the atomic weight of the substance by 4 with formation of a new kind of atom. In a similar manner the instability of the system of electrons within an atom may result in the flying off of an electron. The mass of the electron is so small that the loss of it does not affect the atomic weight of the substance, but changes its chemical and physical properties. The recoil in this case is so small that it is inappreciable.

The emission of β-rays is always accompanied by the emission of γ rays. When it is remembered that the impinging of high velocity electrons upon the atoms of matter gives rise to X-rays, which are similar in character to γ rays, this fact will not cause surprise. The escaping electron has higher velocity than can be produced in the electrons in the discharge tube, and the γ rays produced are more penetrating than the X-rays; also the electron has to traverse part of the atom in its escape.

Radioactive changes.—The study of radioactivity has brought to light a wonderful series of substances whose existence would otherwise never have been suspected. In some cases the amount of the substance in existence is so small that no process of weighing would detect it, and in others the whole life of the substance is extremely short, in one case about 0·00000000001 second. The history of the development

of our knowledge of this series cannot be given here, but a table of these substances is given on p. 203.

Each member of the series has a similar life-history. At a rate which is constant for each substance, and cannot be made to vary by any known means, the atoms change, sometimes by losing an α particle, sometimes by losing an electron. To take the case of radium,—a gas can be separated from it called **radium emanation,** which condenses at $-150°$ C. The radium has lost about 80 per cent. of its activity, but this is found in the emanation. The emanation slowly loses its activity, but the radium regains its activity by the production of emanation which it retains, until eventually the radium attains its original activity ; it is then said to have acquired **radioactive equilibrium.** The amount of emanation in equilibrium with one gramme of radium is called **one curie.** The atom of radium changes to an atom of emanation on emitting one α particle. The fraction of the total number of atoms of a substance breaking up per second is called the **transformation constant** of the substance, and the time required for the decay or transformation of the substance to half its quantity is called the **half-value period** of the substance. The half-value period for radium is 1730 years, while that of the emanation is 3·85 days.

The atom of emanation loses an α particle and becomes a new substance called radium A, whose half-value period is 3·0 minutes, and this loses another α particle and becomes radium B, whose half-value period is 26·8 minutes. Radium B loses a β particle and becomes radium C, which loses a β particle and becomes radium D, then radium E, and then radium F. Radium F loses an α particle and becomes lead, which does not appear to be radioactive and is the end product of this series. It is interesting to note that as the atomic weight of radium is 226, and in its various transformations it loses five α particles each of atomic weight 4, the atomic weight of the last product should be 206. Lead always occurs in minerals containing radium, and the atomic weight of the lead derived from this source has been found to be 206·05.

Radium is only found in minerals containing uranium, and within the last few years the production of radium from uranium by way of the substance ionium has been definitely traced

For the thorium and actinium series the student is referred to the table.

is that of the earth's actual or resultant magnetic field. If this resultant field I be resolved into horizontal and vertical components H and V, it follows that—

$$I^2 = H^2 + V^2$$

also,
$$\tan \theta = \frac{V}{H}$$

where θ is the magnetic dip.

Measurement of magnetic dip.—The instrument used for measuring the magnetic dip is called **a dip circle** (Fig. 196). The magnet is balanced as nearly perfectly as possible about the thin axle which rests on horizontal "knife-edges" made of agate. The angle which the needle makes with the horizontal, or the dip, is observed by means of the vertical circle SS. The instrument can rotate about a vertical axis in order to place the plane of rotation in the magnetic meridian, the position of the instrument being determined with reference to the horizontal scale PP. Accurate centring of the needle with respect to the scale SS is necessary, and for this purpose two V-rests to take the axle can be raised or lowered by turning D. When raised they lift the needle from the knife-edges, and when lowered they let the axle rest on the knife-edges at the centre of the scale.

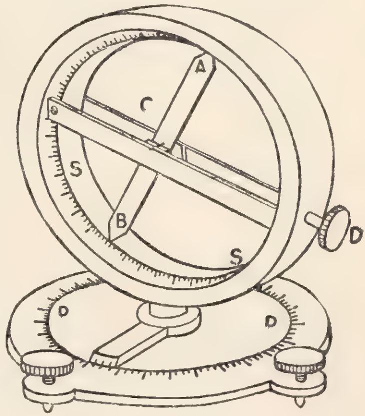


FIG. 196.—Dip circle.

In measuring the dip, the plane of rotation of the dip needle must first be placed in the magnetic meridian. This is done by turning the instrument about its vertical axis until the needle sets vertically. The plane of rotation is then at right angles to the magnetic meridian, and on rotating the instrument through 90° according to the scale PP, the plane of rotation will then be in the magnetic meridian. The reason is that when the plane of

rotation is not at right angles to the meridian, the horizontal component H of the earth's field is able to affect the position of the needle, but when the plane of rotation is at right angles to H (Fig. 197) this component cannot exert a directive effect on the needle, and the vertical component V then causes the needle to set vertically.

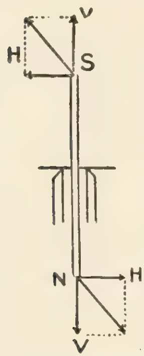


FIG. 197.—Side view of dip needle.

Having placed the plane of rotation in the magnetic meridian there are then four sets of readings to be taken.

(i) The position of both ends of the needle must be read upon the vertical scale, because the axis of rotation may not be at the centre of the scale (Fig. 198 (i)). This causes one end of the needle to indicate too great a dip and the other end too small a dip. By taking the mean, the result is corrected for this error.

(ii) The instrument is rotated through 180° about its vertical axis and the readings (i) repeated. This is because the zero line of the vertical scale may not be truly horizontal (Fig. 198 (ii)).

(iii) Next the needle is turned over on its bearings and (i) and (ii) repeated. The reason for this is that the magnetic axis of the needle may not pass through the two points by means of which the readings are made. Then the points will be as much on one side of the magnetic axis in one position as they are on the other side of the magnetic axis when the needle is reversed (Fig. 198 (iii)). It is, of course, the magnetic axis that sets along the resultant earth's field.

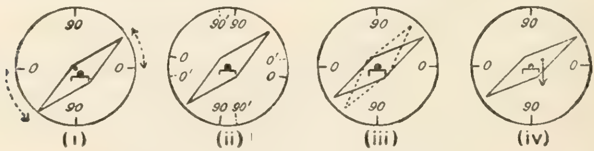


FIG. 198.—Corrections in measuring dip.

(iv) The needle is now taken out of the instrument and remagnetised so that the end which pointed upwards in the previous readings now points downwards, and the readings are all repeated. If the axis of rotation does not pass through the centre of gravity of the needle, that is, if the needle is not truly balanced (Fig. 198 (iv)), one set of readings will indicate too great a dip, and the others will give too small a value.

The mean of the sixteen readings is the value of the dip corrected for all these errors.

Magnetic declination.—A horizontally suspended needle does not point geographically north and south because the magnetic meridian does not as a rule coincide with the geographical meridian. The angle between the two, indicated by α in Fig. 195, is called the **magnetic declination** or, by mariners, the **variation of the compass.** In England, at the present time the magnetic declination is about $15°$ W., that is the compass needle points $15°$ west of north, but the value is getting less by a few minutes of arc each year.

In order to measure the declination, the position of the geographical meridian must be found by astronomical means, and the position of the magnetic meridian by means of a suspended magnet. If the magnet is freely suspended, its magnetic axis comes to rest in the magnetic meridian. But the magnetic axis may not coincide with the geometric axis, so after taking the first reading the magnet is turned over and the position of rest for the magnet is again found, just as for correction (iii) in finding the dip. The bisector of the angle between the two positions of rest is the true direction of the magnetic meridian.

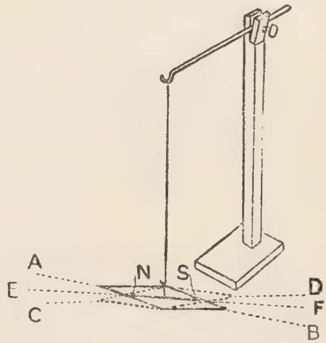
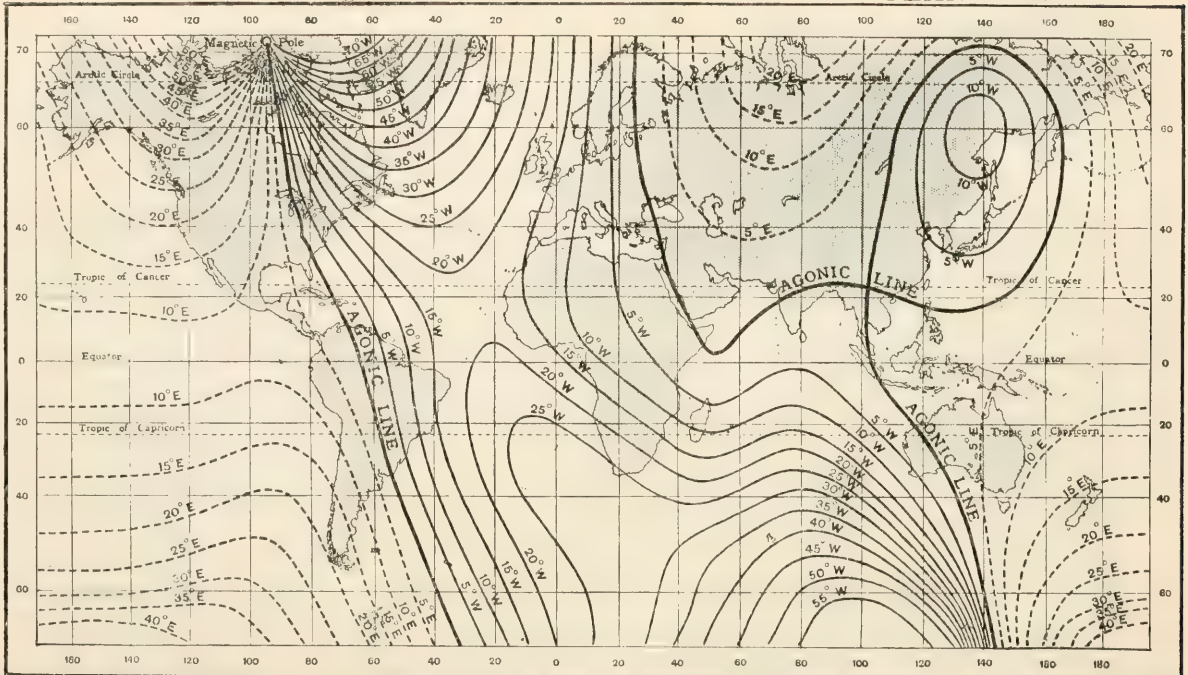


FIG. 199.—Finding the magnetic meridian.

Even when there is a large error it may be corrected in this way; for if a magnetised needle NS (Fig. 199) be covered by two pieces of cardboard cut to the shape of a magnet and suspended by a fine silk fibre, one position of rest AB may be marked on a the bench below the magnet. The magnet is now turned over and suspended from the other side and the position of rest, CD, marked. The bisector EF of the angle between AB and CD is the true direction of the magnetic meridian, and also of the magnetic axis of the needle.

Magnetic elements.—In order to determine completely the magnetic condition at any place upon the earth's surface, it is necessary to measure the declination and two of the

LINES OF EQUAL DECLINATION FOR THE YEAR 1922



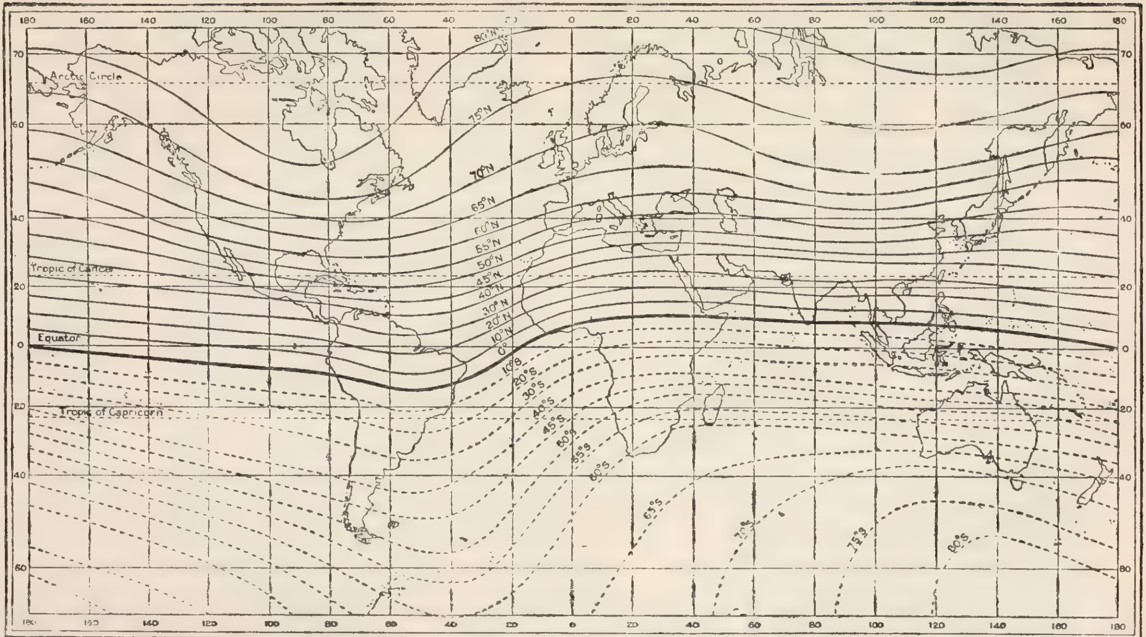
Reproduced by permission from the Admiralty Chart

Cribb & Co. del.

Longmans, Green & Co. London, New York, Toronto, Bombay, Calcutta & Madras

FIG. 200.—Isogonals.

LINES OF EQUAL MAGNETIC DIP



GEORGE PHILIP & SON Ltd

Longmans, Green & Co. London, New York, Bombay & Calcutta.

Reproduced by permission from the Admiralty Chart. FIG. 201.—Isoclinals.

quantities, dip, H, V and I. For convenience, H and the dip are chosen, and the three quantities, declination, dip and H are usually called the **magnetic elements** at the place. V and I can then be calculated as on p. 215. The measurement of the horizontal component H of the earth's magnetic field has been described in Chapter II.

Magnetic maps.—The most convenient representation of the earth's magnetic condition is obtained by drawing lines through all points having the same declination, which are called **isogonals** or **isogonal lines ;** and lines through all points for which the dip is the same, which are called **isoclinals.** In the map (Fig. 200), the isogonals are shown and in Fig. 201 the isoclinals. It will be seen that the isogonals run to the poles, both magnetic and geographic, and there is one line in the American hemisphere for which there is zero declination, that is, the compass points due north. This is called the **agonic line.** There is a similar agonic line running through Eastern Europe, Arabia, and the Indian Ocean, which forms a closed loop in Eastern Asia, called the Siberian Oval.

The isoclinals (Fig. 201) approximate to parallels of latitude, and the N pole of the dipping needle dips in the northern hemisphere, and the S pole in the southern hemisphere. The line of zero dip, or the **magnetic equator,** lies near the geographical equator.

There are two points upon the earth's surface where the dip needle sets vertically. These are called the **magnetic poles.** The magnetic N pole was first found by Sir James Ross in 1831 to be situated longitude 96° 43′ W., latitude 73° 31′ N. The magnetic S pole was found on Sir Ernest Shackleton's expedition in 1909 to be in longitude 155° 16′ E., latitude 72° 25′ S., but the position of both these poles is known to be changing slowly.

More information concerning the isogonals and isoclinals can be obtained from a study of the map than from a description of them.

The earth as a magnet.—It is clear from the distribution of magnetic field around it, that the earth itself is a magnet. This was recognised as long ago as the year 1600 by William Gilbert, who constructed a model of magnetite which represented roughly the magnetic condition of the earth. He was mistaken in considering the magnetic poles to be coincident

with the geographic poles. A closer approximation to the earth's magnetic condition may be obtained by considering a magnet to be situated near the centre of the earth, with its S magnetic pole in the northern hemisphere, and its magnetic axis inclined at an angle of about $17°$ to the axis of rotation. Such a magnet as NS (Fig. 202) would have the magnetic field as shown by dotted lines. The magnetic N pole would be at MNP, where the dip would be $90°$.

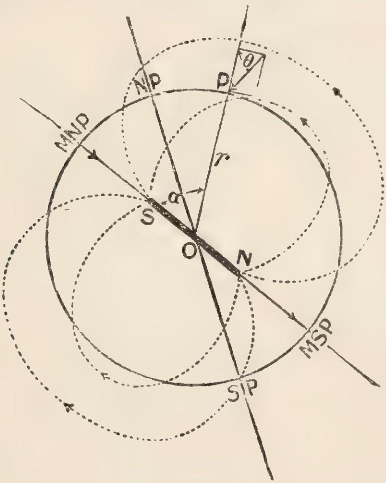


FIG. 202.—Fictitious earth magnet.

At a point such as P we can find an approximate expression for the dip if we consider M to be the magnetic moment of the magnet NS, r the radius of the earth and α the angle made between OP and the axis of the magnet. If the magnetic moment M be resolved into a component $M \cos \alpha$ in the direction OP and $M \sin \alpha$ at right angles to OP, the former would give a vertical component $\frac{2M \cos \alpha}{r^3} = V$, at P (p. 15) and the latter a horizontal component $\frac{M \sin \alpha}{r^3} = H$. Now if θ is the dip—

$$\tan \theta = \frac{V}{H} = \frac{2M \cos \alpha}{r^3} \cdot \frac{r^3}{M \sin \alpha}$$

$$= 2 \cot \alpha$$

Although the interior magnet NS gives a field which strongly resembles the actual magnetic field of the earth, still the actual irregularities are so great that it is obvious that no simple magnetic model of the earth can be given. In spite of this fact, the hypothetic interior magnet affords a simple method of visualising the broad characteristics of the earth's field.

Variations in the earth's magnetic field.—The magnetic condition of the earth is continually changing and the changes occurring range themselves under several distinct heads.

(i) **Secular variation.**—In about 960 years, the declination at every place undergoes a cycle of change. Thus in the year 1659 the magnetic compass at London pointed due north, that is the declination was zero. In 1823 it reached a maximum westerly declination of $24\frac{1}{2}°$. At the present time the declination is about 15° W. and it is decreasing slowly. The older records are uncertain, but there is reason to believe that in about the year 2139 the declination will again be zero, having completed a half cycle, after which it will be easterly. The secular change may be very fairly represented by a rotation of the magnetic poles around the geographic poles in a circle of 17° radius. Thus if P (Fig. 203) is the geographic pole, the circle travelled by the magnetic pole is shown. In 1580, the earliest observation on record, the declination at London was 11° 15′ E., in 1600 it was 4° 5′ E., and in 1659 it was zero. It was pointed out by Lord Kelvin that the magnetic system of the earth was rotating in a westerly direction, making a complete cycle in about 960 years.

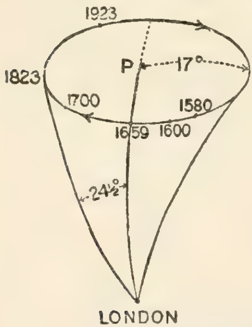


FIG. 203.—Secular variation.

(ii) **Daily variation.**—The daily variation of the earth's magnetic field is small and can only be measured at fixed observatories where very delicate recording instruments can be employed. A mirror attached to the suspended magnet produces an optical image of a source of light, upon a drum of sensitised paper. As the drum rotates, the image of the spot of light leaves a track, from which the variation in the

declination or dip can be derived. Fig. 204 represents the normal daily variations recorded at the Kew observatory. The curve A represents the variation $\delta\theta$ in the declination, the maximum of 5′ W. at about 1·0 p.m. The curve B

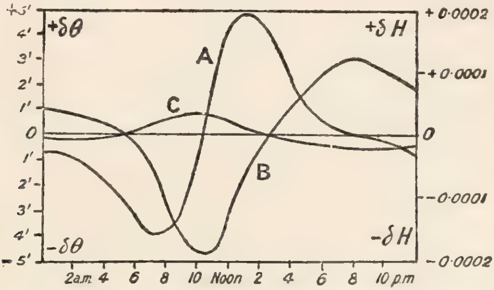


FIG. 204.—Daily variations in earth's magnetic field.

gives the variation δH in the horizontal intensity of the earth's field, and C gives the variation in dip to the same scale as A. The daily variations are of three distinct types given in Fig. 205. O is the variation for an ordinary day, D for a disturbed day or day of unusually great disturbance, and Q for a quiet day.

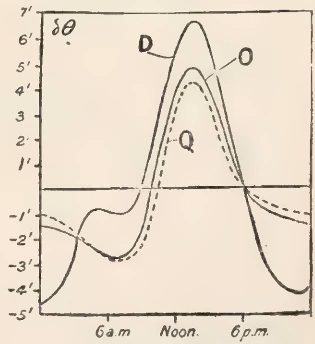


FIG. 205.—Classification of daily variations.

(iii) **Annual variation.**—This is extremely small and amounts to a variation in the declination and occurs simultaneously in opposite directions in the northern and southern hemispheres. At London a maximum easterly variation of about $2\frac{1}{4}'$ occurs in August, with a corresponding westerly variation in February.

(iv) **Eleven-year period.**—The magnitude of the daily variation undergoes a cycle occupying a period of eleven years, and its maximum coincides with maximum sun-spot frequency, which also goes through an eleven-year period of change. The maxima occurred in 1861, 1872, 1883, 1894, etc.

(v) **Magnetic storms.**—At least part of the daily variation in the earth's magnetic field is due to electric currents in the conducting layers of the upper atmosphere of the earth. Any disturbance of this layer will cause sudden and unforeseen disturbances in the magnetic field, resulting in irregular disturbances of the compass over large areas. These are called **magnetic storms**, and frequently occur during displays of the aurora borealis. This explanation is consistent with the fact that large quantities of electrons emitted with high velocity by the sun, on entering the earth's atmosphere would considerably increase its conductivity.

Exercises on Chapter XVI

1. Define the terms "declination" and "dip" and describe how the declination may be measured.

2. Describe the process of finding the magnetic dip, and the corrections necessary in obtaining an accurate result.

3. Give the relation between dip, H and V.

If the horizontal component of the earth's magnetic field at a certain place is 0·18 C.G.S. unit and the dip is 70°, find the vertical component and the resultant intensity of the earth's field.

4. A dip needle makes 30 vibrations per minute at a place where the dip is 75°, and 40 vibrations at a place where the dip is 65°. What is the ratio of the horizontal components of the earth's magnetic field at the two places?

5. Describe and give reasons for the method of setting the dip needle with its plane of rotation in the magnetic meridian.

6. Give a general account of the course of the isogonals and isoclinals upon the earth's surface.

7. Give an account of the secular variation of the earth's magnetic field.

8. Describe the general form of the daily variations of the earth's magnetic field.

9. Enumerate and describe briefly the chief variations of the earth's magnetic field.

10. Describe a simple device of an interior magnet for representing roughly the earth's magnetic condition.

11. At a point on the earth 30° from the magnetic pole, find an approximate value for the dip.

CHAPTER XVII

MAGNETIC PROPERTIES OF MATERIALS

Theories of magnetisation.—The very early theories to account for magnetic properties of iron and steel are not of great importance, but the theory of Weber, that the molecules of magnetic substances are themselves permanent magnets and that the act of magnetisation consists in setting them in one direction, is the foundation of the present molecular theory of magnetisation. This has already been mentioned (p. 9), but it leaves one fact unexplained, namely, that the application of a moderate magnetic field does not turn all the molecular magnets at once into the same direction and so

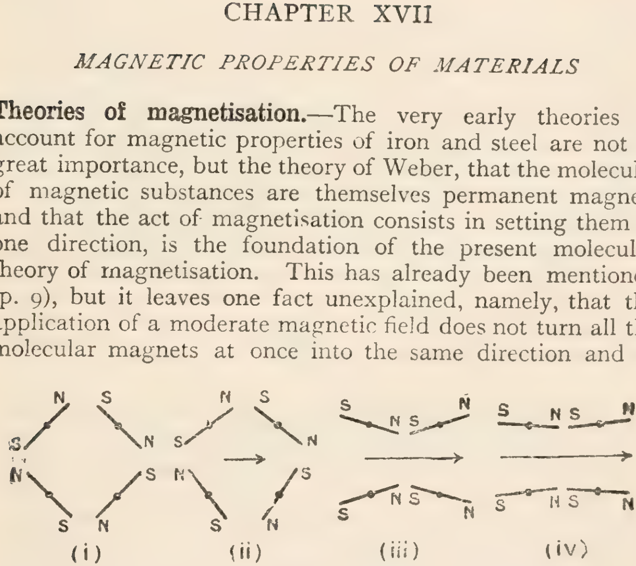


FIG. 206.—Ewing's molecular theory of magnetisation.

produce saturation immediately. It was then suggested that there is friction opposing the rotation of the elementary magnets, but Sir J. A. Ewing has shown that the magnetic interaction of the molecular magnets themselves dispenses with the necessity of supposing friction to be present.

For, if a group of four simple magnets (Fig. 206 (i)), each free to rotate about its own axis, be imagined, the group will arrange itself as shown, because of the attractions of the N for the S poles, and there will be symmetry in the arrangement. Now imagine a weak magnetic field to be applied (ii), the magnets will turn slightly in the direction of the field, but the forces between the poles

are still strong enough to prevent the breaking up of the group. On further increasing the field, a time will come when the group becomes unstable and the magnets will swing round into the position (iii). Any further increase in the field can only result in a further alignment (iv), as the condition of saturation is nearly reached. In Fig. 207 a curve is drawn indicating the intensity of magnetisation in the four stages (i), (ii), (iii) and (iv) of the group of molecular magnets. When it is remembered that a magnetic material consists of an infinite variety of groups of all degrees of stability it will be realised that some groups will break up before others and the angles of the curve of Fig. 207 will be smoothed off, and a curve very similar to that for an actual material (O A B, Fig. 215) will be obtained. Although Ewing has modified his model recently, the above explanation remains essentially true.

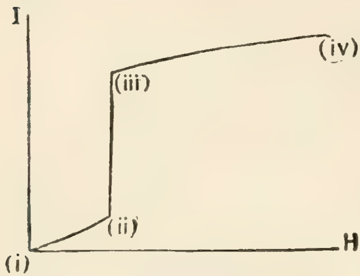


FIG. 207.—Ideal magnetisation curve.

Intensity of magnetisation.—In studying the magnetic properties of materials it is necessary to define several new quantities. Thus the **intensity of magnetisation** of a specimen is defined as the **magnetic moment per unit volume.** Or, for a uniformly magnetised specimen, the intensity of magnetisation is the ratio of magnetic moment to volume.

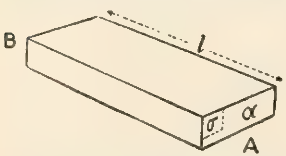


FIG. 208.—Magnetised iron bar.

If Fig. 208 represents a rectangular block of iron uniformly magnetised with end A, say, N pole, and B, S pole, then—

$$\text{Intensity of magnetisation} = \frac{\text{magnetic moment}}{\text{volume}}$$

$$= \text{I}$$

Now let a be the area of each end and σ the amount of pole per square centimetre of each end, $a\sigma$ is then the

amount of pole at each end, and if l is the length of the bar—

$$\text{Magnetic moment} = la\sigma \text{ (p. 13)}$$

$$\text{Volume} = la$$

$$\therefore \ I = \frac{la\sigma}{la} = \sigma$$

Hence the intensity of magnetisation may also be defined as the **amount of pole per square centimetre of the end of the bar.**

Magnetic susceptibility.—The intensity of magnetisation produced in any magnetic material depends upon the nature of the material and the strength of the magnetic field to which the magnetisation is due. The ratio of intensity of magnetisation to the strength of field which produces it is called the **magnetic susceptibility** (k) of the material.

$$\text{Magnetic susceptibility} = \frac{\text{Intensity of magnetisation}}{\text{Magnetising field}}$$

$$k = \frac{I}{H}$$

or,

$$I = kH$$

It will be seen later that k is not a simple quantity, as I is not proportional to H, but varies in a very complex manner with it (p. 235). Hence k varies with the strength of field and also with the history of the specimen.

Magnetic permeability.—On referring to our early work on p. 10, it will be remembered that the unit of magnetic pole was chosen so that the force between the unit poles situated one centimetre apart **in vacuo** is one dyne. At the time, the medium in which the poles are situated was not mentioned, it was assumed to be air, and the result is almost exactly the same as when the poles are in a vacuum. But for many media the result would not be the same; if the poles are separated by iron, nickel or cobalt the force between them would be very much less, if separated by bismuth slightly greater, than when the poles are situated in air. The simple equation of p. 10 must therefore be modified, and it will now be written—

$$F = \frac{m_1 m_2}{\mu d^2} \text{ dynes}$$

The quantity μ is called the **magnetic permeability** of the

medium in which the poles are situated. It is the quantity which applies the correction to the force equation in order to render it valid for all media in which the magnetic poles may be situated. The magnetic permeability is therefore taken to be unity for vacuum, and it is very nearly unity for air but is very great for iron, nickel and cobalt, the ferro-magnetic metals (p. 21).

Magnetic induction.—It is now necessary to define more exactly the quantity known as magnetic induction (B) which has already been mentioned (p. 20). Consider any point in any medium, and let the strength of magnetic field at the point be represented by H, and defined as the force which would act upon a magnetic pole of unit strength placed at the point. Further, let the permeability μ of the medium at the point be defined from the relation $F=\frac{m_1 m_2}{\mu d^2}$ dynes ; then the product μH is called the **magnetic induction** at the point considered.

Thus, magnetic induction=magnetic field × permeability

$$B=\mu H$$

It will be noticed that the strength of field at distance d from a pole of strength m is—

$$H=\frac{m}{\mu d^2}$$

instead of m/d^2 as given on p. 12.

Also, the magnetic induction at distance d from a pole of strength m is—

$$B=\mu H=\mu\frac{m}{\mu d^2}=\frac{m}{d^2}$$

that is, the magnetic induction at any point in the neighbourhood of a magnetic pole is independent of the nature of the medium in which the pole is situated.

Magnetic induction, just like magnetic field, may be represented by lines (p. 20). Thus if the field be represented by H lines per square centimetre, there will be μH or B lines of induction per square centimetre. In air and in all media in which μ is practically unity, B and H are the same numerically and it is for this reason that H is frequently referred to instead of B. The distinction is only of importance in dealing with media such as iron, where μ is not unity.

For example, the force on a current (p. 133) is really B*i* dynes

per centimetre of conductor, not H*i*, and the electromotive force in a conductor is equal to the **number of lines of magnetic induction cut per second,** not the number of lines of **force** (p. 140).

Gauss's law.—For the determination of the relation between magnetic permeability and susceptibility it is necessary to quote here an extension of the law of inverse squares, due to Gauss. The law will not be proved here [1] as more mathematics is required than we can assume, but it is simple in result and very important. It expresses the fact that the amount of magnetic pole situated at a point can be represented by the state of the medium surrounding the point. The only quantity which depends upon the strength of pole alone, being independent of the nature of the medium, is the magnetic induction (p. 232). We may state Gauss's law most conveniently as follows :—

The number of lines of magnetic induction crossing any closed surface surrounding a magnetic pole of strength m is equal to $4\pi m$. The lines are all directed outwards if the pole is a N pole and inwards if a S pole, and the law is true for any number of poles situated within the surface. Thus m is the algebraic sum of all the poles within the closed surface.

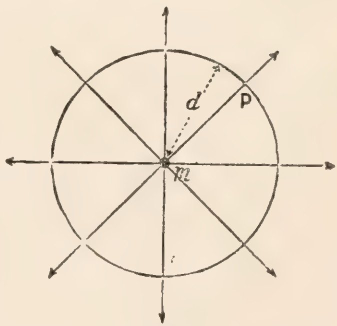


FIG. 209.—Illustration of Gauss's law.

To derive Gauss's law from the inverse square law is not easy, but the reverse process is very simple. Consider a N pole of strength m (Fig. 209). Let us find the strength of magnetic field H at the point P, distant d cm. from m. Through P describe a sphere with m as centre. From symmetry we know that the magnetic field has the same strength all over this sphere. The magnetic induction at P is μH, so that μH lines of magnetic induction cross 1 sq. cm. at P. Therefore, since the area of the sphere is $4\pi d^2$ sq. cm. the whole number of lines of induction crossing the sphere is $4\pi d^2\mu H$.

But by Gauss's law, this number is $4\pi m$.

$$\therefore \; 4\pi d^2\mu H = 4\pi m$$

$$H = \frac{m}{\mu d^2}$$

[1] See "Electricity and Magnetism for Advanced Students," by S. G. Starling. Messrs. Longmans, Green & Co.

a result already given on p. 228, which is an expression of the inverse square law.

Iron in magnetic field.—Suppose a bar of iron, originally unmagnetised, to be situated in a uniform magnetic field of strength H (Fig. 210). The iron becomes magnetised, poles being developed at N and S. There will now be a magnetic field developed by the iron magnet, and this is indicated by the dotted lines. In some places this field is added to the original field H, as at A and B, and at other places, such as C and D, the field due to the magnet is in opposition to the original field H. If the resultant of the two fields be obtained and this resultant field drawn, it will be of the form shown in Fig. 211. At great distances from the bar the original field is undisturbed, but near the magnet it will be seen that the resultant field is concentrated upon it. This tendency of the lines to concentrate upon the specimen is characteristic of the materials for which the permeability is greater than unity, and is very marked in the case of the ferro-magnetic metals, but only slightly for such metals as platinum and aluminium, for which μ is very little greater

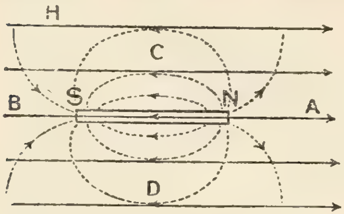


FIG. 210.—Iron in uniform magnetic field.

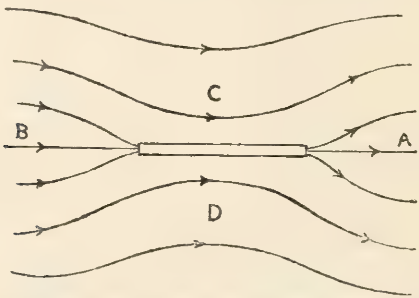


FIG. 211.—Resultant of field due to iron and original field.

than unity (p. 233). Such metals are said to be **paramagnetic.** They have also the property of being attracted from the weaker to the stronger parts of a magnetic field and of setting, when suspended, with the long axis of the specimen parallel to the field. It is by these feeble effects that the susceptibility and permeability have been measured. Another group of metals, such as copper, gold, silver, bismuth, etc., and water have a permeability slightly less than unity and the lines of Fig. 211 appear pushed away from the specimen, which is said to be **diamagnetic.** Diamagnetic bodies are driven from the stronger to the weaker parts of a magnetic field, and when suspended set with their longer axis at right angles to the magnetic field.

Demagnetising effect.—Another effect may be deduced from Fig. 210, for the poles N and S produce, as we have seen, a field which in the middle of the magnet itself is directed from N to S. This field is in such a direction that it tends to demagnetise the magnet itself. Thus all magnets having poles produce magnetic fields which, within the magnets, have a tendency to produce demagnetisation.

It is for this reason that **keepers** are employed with bar and horseshoe magnets when not in use. The keeper is a piece of soft iron placed in such a way that it is in contact with a N and a S pole (Fig. 212). In the keeper, poles are produced which are, of course, of opposite kind to poles with which they are in contact. It is clear that if the poles N and S produce a **demagnetising** field, the poles N′ S′ produce a **magnetising** field. Thus the effect of the keeper is to remove the demagnetising field of the poles of the steel magnets. Also, the demagnetising effect is much less in a long, thin magnet than in a short one, for the poles in the former case are small and are far removed from the central portion of the magnet. It is for this reason that it is impossible to produce intense magnetisation in a thin sheet of iron in a direction at right angles to the plane of the sheet. The poles on the surface produce a field in the interior in the opposite direction to, and nearly as strong as, the magnetising field. This is the extreme

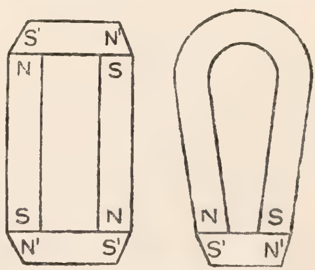


FIG. 212.—Magnets with keepers.

case, and for an iron sheet, the interior resultant field producing magnetisation may be only $\frac{1}{1000}$ of the original magnetising field.

Magnetic induction in magnetic materials.—If the bar in Fig. 210 is fairly long we may neglect the demagnetising field in its interior and consider the magnetising field H to be undiminished. In order to find the number of lines of induction passing down the iron, take the cross-section as a sq. cm. and the intensity of magnetisation I. Then the total amount of pole is $\mathrm{I}a$ (p. 227) and this pole is at or near the end of the bar. Even if it is not entirely at the end, it follows from Gauss's law that $4\pi\mathrm{I}a$ lines of induction radiate from it. Similarly, lines run from molecule to molecule down the bar (Fig. 213). The closed surface through which these lines pass must, of course, only include the pole at the

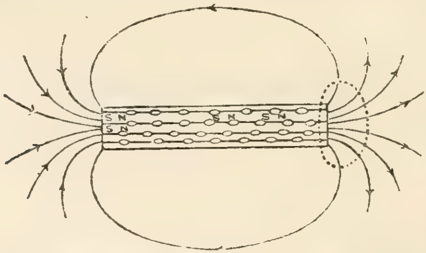


FIG. 213.—Magnetic induction in interior of iron bar.

end of the bar as shown in dotted line in Fig. 213. Thus the number of lines of induction passing down the bar is $4\pi\mathrm{I}a$ due to the magnetisation of the iron, and to these must be added $\mathrm{H}a$ lines of the original field, making in all $\mathrm{B}a$ lines where B is the value of the induction within the metal.

Thus, $$\mathrm{B}a=\mathrm{H}a+4\pi\mathrm{I}a$$

or, $$\mathrm{B}=\mathrm{H}+4\pi\mathrm{I}$$

This important relation shows that B, I and H are not independent quantities. If two be known, the other one can be calculated.

Again, if the equation be divided through by H—

$$\frac{\mathrm{B}}{\mathrm{H}}=\frac{\mathrm{H}}{\mathrm{H}}+4\pi\frac{\mathrm{I}}{\mathrm{H}}$$

or, $$\mu=1+4\pi k$$

Thus μ and k are closely related. For unmagnetisable materials $k=0$, and therefore $\mu=1$. For paramagnetic materials $\mu>1$, therefore k is a positive, and for diamagnetic materials $\mu<1$, therefore k is negative. For these materials k is always a very small quantity, so that μ differs very little from unity.

Aluminium	$k=+6{\cdot}5\times10^{-6}$
Bismuth	$-1{\cdot}4\times10^{-6}$
Copper	$-0{\cdot}087\times10^{-6}$
Gold	$-0{\cdot}15\times10^{-6}$
Platinum	$+1{\cdot}32\times10^{-6}$
Silver	$-0{\cdot}2\times10^{-6}$
Water	$=0{\cdot}77\times10^{-6}$

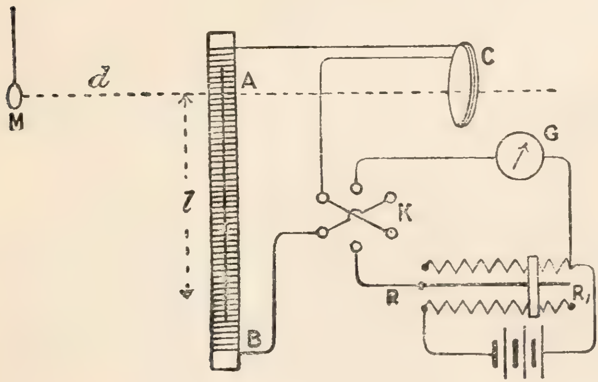


FIG. 214.—Ewing's magnetometer.

Measurement of intensity of magnetisation.—The most direct method of studying the dependence of intensity of magnetisation upon magnetising field is due to Sir J. A. Ewing, and is known as the **magnetometer method.**

The specimen to be examined must be in the form of a long, fairly thin wire AB (Fig. 214), so that the demagnetising effect of the poles (p. 231) may be small or negligible. The specimen is placed in a vertical solenoid carrying a current, so that it is situated in a magnetising field $H=4\pi ni$, where n is the number of turns per centimetre of the solenoid and i is the current in absolute measure, determined by the ammeter G. The vertical component

of the earth's magnetic field will cause some disturbance, and this can be removed by using a second solenoid wound outside the first and maintaining a suitable current in it. This refinement is not always necessary.

In order to measure the intensity of magnetisation of the specimen, its upper pole A is placed on a level with a magnetometer needle M and the deflection of the spot of light measures the angle of deflection, as in the case of the reflecting galvanometer (p. 31). The strength of pole at the end of the specimen is $\mathrm{I}\alpha$ situated d cm. from the needle and produces magnetic field $\mathrm{I}\alpha/d^2$, so that if the pole is east or west of the needle and the earth's horizontal field is H′—

$$\frac{\mathrm{I}\alpha}{d^2}=\mathrm{H}'\tan\theta$$

where θ is the deflection (p. 17)

or,
$$\mathrm{I}=\frac{d^2\mathrm{H}'}{\alpha}\tan\theta$$

If the lower pole B is not so far removed that its effect on the magnetometer needle is negligible, the expression for I and θ becomes—

$$\mathrm{I}\alpha\left\{\frac{1}{d^2}-\frac{d}{(d^2+l^2)^{\frac{3}{2}}}\right\}=\mathrm{H}'\tan\theta$$

The field H′ may not be known, in fact it may be an artificial field produced by a controlling magnet (p. 29). In this case it is necessary to find its value. This is done by means of the vertical circular coil C of n_1 turns. If, after completing the measurements with the specimen, the solenoid is cut out and a current i_1 passed through C, this produces a magnetic field $\frac{2\pi n_1 a^2 i_1}{(a^2+x^2)^{\frac{3}{2}}}$ at the magnetometer needle, where a is the radius of the coil C, and x its distance from the needle. The deflection θ_1 is then observed—

and,
$$\frac{2\pi n_1 a^2 i_1}{(a^2+x^2)^{\frac{3}{2}}}=\mathrm{H}'\tan\theta_1$$

from which H′ can be found. It is then possible to calculate I, the intensity of magnetisation of the specimen.

There is one point which must be noticed, which is, that the solenoid itself is equivalent to a magnet (p. 23) and the deflections may be partly due to this effect and partly due to the specimen. To eliminate this disturbance the coil C is connected in series with the solenoid while the observations with the specimen are being made, its distance from the magnetometer having been previously adjusted, with the specimen removed, until the field due to C at

the magnetometer is equal and opposite to that due to the solenoid. The use of the compensating coil C precludes the necessity of making any allowance for the disturbing effect of the solenoid upon the magnetometer.

Cycle of magnetisation.—The measurements with the magnetometer are begun by using a small current, regulated by the rheostat RR_1 (Fig. 214) and observing successive values of i and θ, from which H and I can be calculated. These may be plotted, giving the curve OAB (Fig. 215). The intensity of magnetisation increases slowly at first, then rapidly, and finally very slowly, as saturation is approached. Having attained the point B, the current is now decreased

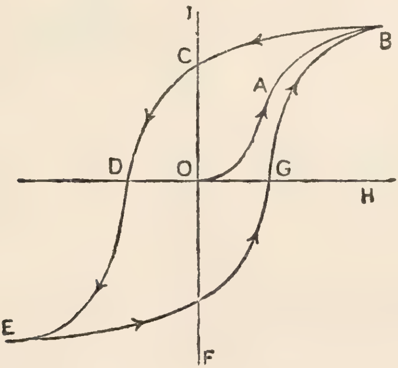


FIG. 215.—Cycle of magnetisation.

and the observations continued. This gives the falling curve BC. By means of the reversing key K, the direction of the current is changed and with negative values of H the field is again increased, giving the curve CDE. On again diminishing, reversing, and increasing the field the curve EFGB is obtained, thus completing the cycle.

Hysteresis.—The first study of the cycle of magnetisation was due to Sir James Ewing. The cycle of Fig. 215 is characteristic of the ferro-magnetic materials, iron, nickel and cobalt. It will be seen that on traversing the cycle, the zero of magnetisation (D and G) always occurs at a later point in the cycle than the zero of magnetising field (C and F).

For this reason the name **hysteresis** is given to this phenomenon, being derived from the Greek word meaning "to lag." The value of OC, the intensity of magnetisation remaining when the magnetising field is reduced to zero, is called the **residual magnetism.** The value OD of the reverse magnetising field required to reduce the intensity of magnetisation to zero is called the **coercive force** for the material.

It should be noted that work is necessary to take a specimen round a cycle of magnetisation, and this work appears as heat in the specimen. The amount of work per cubic centimetre of specimen which is converted into heat during one cycle of magnetisation is equal to the area of the hysteresis curve plotted to absolute scale of I and H.[1]

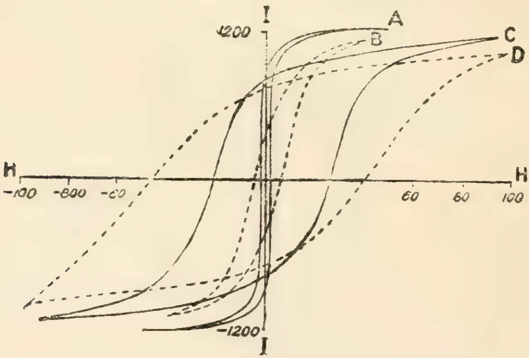


FIG. 216.—Hysteresis cycles for iron and steel.

Iron and steel.—The cycles of magnetisation for iron and steel differ in certain details. That for iron is always smaller in area and more angular than that for steel. Curve A (Fig. 216) taken from Sir James Ewing's results is for annealed soft iron wire, while B is for the same wire hardened by stretching. C is for annealed pianoforte steel wire, while D is for the same wire rendered glass-hard. It will be noticed that for steel, the residual magnetism is less, and the coercive force greater, than for iron. Nickel and cobalt give similar curves to iron and steel, but the saturation value

[1] See "Electricity and Magnetism for Advanced Students," by S. G. Starling Messrs. Longmans, Green & Co.

of I for nickel is only about one-third of that for iron, while cobalt has a value approximating to that for iron. The residual magnetism and coercive force are smaller for nickel and cobalt than for iron.

Exercises on Chapter XVII

1. Give a short account of the molecular theory of magnetisation.

2. Define "intensity of magnetisation."

A round bar of iron of diameter 0·6 cm. and length 8 cm. has an intensity of magnetisation 40. What is its magnetic moment?

3. Find the strength of magnetic field at a distance of 60 cm. from the mid-point of the bar, if the length is 12 cm., area of section 2 sq. cm., and intensity of magnetisation 200, the point being on a line bisecting the magnet at right angles.

4. Define "magnetic susceptibility" and "magnetic permeability." How are they related to each other?

5. Prove the relation for the connection between B, H and I within a magnetic material.

6. A piece of iron wire in a magnetic field of strength 30 is magnetised to intensity 300. What is the permeability of the material?

7. The magnetic susceptibility of platinum being 22×10^{-6} and of bismuth $-13{\cdot}2 \times 10^{-6}$, find their magnetic permeabilities.

8. What are paramagnetic and diamagnetic substances? How may they be distinguished experimentally?

9. Explain the effect of demagnetisation, and the use of keepers for permanent magnets.

10. Describe with sketches the effect upon the form of a magnetic field of putting a piece of a ferro-magnetic substance and a piece of a diamagnetic substance into the field.

11 Describe an experiment for the determination of the variations of intensity of magnetisation with magnetising field for a piece of iron wire.

12. What is hysteresis? Indicate the chief points in a cycle of magnetisation.

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
10	0000	0043	0086	0128	0170						4	9	13	17	21	25	30	34	38
						0212	0253	0294	0334	0374	4	8	12	16	20	24	28	32	37
11	0414	0453	0492	0531	0569						4	8	12	15	19	23	27	31	35
						0607	0645	0682	0719	0755	4	7	11	15	19	22	26	30	33
12	0792	0828	0864	0899	0934	0969					3	7	11	14	18	21	25	28	32
							1004	1038	1072	1106	3	7	10	14	17	20	24	27	31
13	1139	1173	1206	1239	1271						3	7	10	13	16	20	23	26	30
						1303	1335	1367	1399	1430	3	7	10	12	16	19	22	25	29
14	1461	1492	1523	1553							3	6	9	12	15	18	21	24	28
					1584	1614	1644	1673	1703	1732	3	6	9	12	15	17	20	23	26
15	1761	1790	1818	1847	1875	1903					3	6	9	11	14	17	20	23	26
							1931	1959	1987	2014	3	5	8	11	14	16	19	22	25
16	2041	2068	2095	2122	2148						3	5	8	11	14	16	19	22	24
						2175	2201	2227	2253	2279	3	5	8	10	13	15	18	21	23
17	2304	2330	2355	2380	2405	2430					3	5	8	10	13	15	18	20	23
							2455	2480	2504	2529	2	5	7	10	12	15	17	19	22
18	2553	2577	2601	2625	2648						2	5	7	9	12	14	16	19	21
						2672	2695	2718	2742	2765	2	5	7	9	11	14	16	18	21
19	2788	2810	2833	2856	2878						2	4	7	9	11	13	16	18	20
						2900	2923	2945	2967	2989	2	4	6	8	11	13	15	17	19
20	3010	3032	3054	3075	3096	3118	3139	3160	3181	3201	2	4	6	8	11	13	15	17	19
21	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404	2	4	6	8	10	12	14	16	18
22	3424	3444	3464	3483	3502	3522	3541	3560	3579	3598	2	4	6	8	10	12	14	15	17
23	3617	3636	3655	3674	3692	3711	3729	3747	3766	3784	2	4	6	7	9	11	13	15	17
24	3802	3820	3838	3856	3874	3892	3909	3927	3945	3962	2	4	5	7	9	11	12	14	16
25	3979	3997	4014	4031	4048	4065	4082	4099	4116	4133	2	3	5	7	9	10	12	14	15
26	4150	4166	4183	4200	4216	4232	4249	4265	4281	4298	2	3	5	7	8	10	11	13	15
27	4314	4330	4346	4362	4378	4393	4409	4425	4440	4456	2	3	5	6	8	9	11	13	14
28	4472	4487	4502	4518	4533	4548	4564	4579	4594	4609	2	3	5	6	8	9	11	12	14
29	4624	4639	4654	4669	4683	4698	4713	4728	4742	4757	1	3	4	6	7	9	10	12	13
30	4771	4786	4800	4814	4829	4843	4857	4871	4886	4900	1	3	4	6	7	9	10	11	13
31	4914	4928	4942	4955	4969	4983	4997	5011	5024	5038	1	3	4	6	7	8	10	11	12
32	5051	5065	5079	5092	5105	5119	5132	5145	5159	5172	1	3	4	5	7	8	9	11	12
33	5185	5198	5211	5224	5237	5250	5263	5276	5289	5302	1	3	4	5	6	8	9	10	12
34	5315	5328	5340	5353	5366	5378	5391	5403	5416	5428	1	3	4	5	6	8	9	10	11
35	5441	5453	5465	5478	5490	5502	5514	5527	5539	5551	1	2	4	5	6	7	9	10	11
36	5563	5575	5587	5599	5611	5623	5635	5647	5658	5670	1	2	4	5	6	7	8	10	11
37	5682	5694	5705	5717	5729	5740	5752	5763	5775	5786	1	2	3	5	6	7	8	9	10
38	5798	5809	5821	5832	5843	5855	5866	5877	5888	5899	1	2	3	5	6	7	8	9	10
39	5911	5922	5933	5944	5955	5966	5977	5988	5999	6010	1	2	3	4	5	7	8	9	10
40	6021	6031	6042	6053	6064	6075	6085	6096	6107	6117	1	2	3	4	5	6	8	9	10
41	6128	6138	6149	6160	6170	6180	6191	6201	6212	6222	1	2	3	4	5	6	7	8	9
42	6232	6243	6253	6263	6274	6284	6294	6304	6314	6325	1	2	3	4	5	6	7	8	9
43	6335	6345	6355	6365	6375	6385	6395	6405	6415	6425	1	2	3	4	5	6	7	8	9
44	6435	6444	6454	6464	6474	6484	6493	6503	6513	6522	1	2	3	4	5	6	7	8	9
45	6532	6542	6551	6561	6571	6580	6590	6599	6609	6618	1	2	3	4	5	6	7	8	9
46	6628	6637	6646	6656	6665	6675	6684	6693	6702	6712	1	2	3	4	5	6	7	7	8
47	6721	6730	6739	6749	6758	6767	6776	6785	6794	6803	1	2	3	4	5	5	6	7	8
48	6812	6821	6830	6839	6848	6857	6866	6875	6884	6893	1	2	3	4	4	5	6	7	8
49	6902	6911	6920	6928	6937	6946	6955	6964	6972	6981	1	2	3	4	4	5	6	7	8
50	6990	6998	7007	7016	7024	7033	7042	7050	7059	7067	1	2	3	3	4	5	6	7	8

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
51	7076	7084	7093	7101	7110	7118	7126	7135	7143	7152	1	2	3	3	4	5	6	7	8
52	7160	7168	7177	7185	7193	7202	7210	7218	7226	7235	1	2	2	3	4	5	6	7	7
53	7243	7251	7259	7267	7275	7284	7292	7300	7308	7316	1	2	2	3	4	5	6	6	7
54	7324	7332	7340	7348	7356	7364	7372	7380	7388	7396	1	2	2	3	4	5	6	6	7
55	7404	7412	7419	7427	7435	7443	7451	7459	7466	7474	1	2	2	3	4	5	5	6	7
56	7482	7490	7497	7505	7513	7520	7528	7536	7543	7551	1	2	2	3	4	5	5	6	7
57	7559	7566	7574	7582	7589	7597	7604	7612	7619	7627	1	2	2	3	4	5	5	6	7
58	7634	7642	7649	7657	7664	7672	7679	7686	7694	7701	1	1	2	3	4	4	5	6	7
59	7709	7716	7723	7731	7738	7745	7752	7760	7767	7774	1	1	2	3	4	4	5	6	7
60	7782	7789	7796	7803	7810	7818	7825	7832	7839	7846	1	1	2	3	4	4	5	6	6
61	7853	7860	7868	7875	7882	7889	7896	7903	7910	7917	1	1	2	3	4	4	5	6	6
62	7924	7931	7938	7945	7952	7959	7966	7973	7980	7987	1	1	2	3	3	4	5	6	6
63	7993	8000	8007	8014	8021	8028	8035	8041	8048	8055	1	1	2	3	3	4	5	5	6
64	8062	8069	8075	8082	8089	8096	8102	8109	8116	8122	1	1	2	3	3	4	5	5	6
65	8129	8136	8142	8149	8156	8162	8169	8176	8182	8189	1	1	2	3	3	4	5	5	6
66	8195	8202	8209	8215	8222	8228	8235	8241	8248	8254	1	1	2	3	3	4	5	5	6
67	8261	8267	8274	8280	8287	8293	8299	8306	8312	8319	1	1	2	3	3	4	5	5	6
68	8325	8331	8338	8344	8351	8357	8363	8370	8376	8382	1	1	2	3	3	4	4	5	6
69	8388	8395	8401	8407	8414	8420	8426	8432	8439	8445	1	1	2	2	3	4	4	5	6
70	8451	8457	8463	8470	8476	8482	8488	8494	8500	8506	1	1	2	2	3	4	4	5	6
71	8513	8519	8525	8531	8537	8543	8549	8555	8561	8567	1	1	2	2	3	4	4	5	5
72	8573	8579	8585	8591	8597	8603	8609	8615	8621	8627	1	1	2	2	3	4	4	5	5
73	8633	8639	8645	8651	8657	8663	8669	8675	8681	8686	1	1	2	2	3	4	4	5	5
74	8692	8698	8704	8710	8716	8722	8727	8733	8739	8745	1	1	2	2	3	4	4	5	5
75	8751	8756	8762	8768	8774	8779	8785	8791	8797	8802	1	1	2	2	3	3	4	5	5
76	8808	8814	8820	8825	8831	8837	8842	8848	8854	8859	1	1	2	2	3	3	4	5	5
77	8865	8871	8876	8882	8887	8893	8899	8904	8910	8915	1	1	2	2	3	3	4	4	5
78	8921	8927	8932	8938	8943	8949	8954	8960	8965	8971	1	1	2	2	3	3	4	4	5
79	8976	8982	8987	8993	8998	9004	9009	9015	9020	9025	1	1	2	2	3	3	4	4	5
80	9031	9036	9042	9047	9053	9058	9063	9069	9074	9079	1	1	2	2	3	3	4	4	5
81	9085	9090	9096	9101	9106	9112	9117	9122	9128	9133	1	1	2	2	3	3	4	4	5
82	9138	9143	9149	9154	9159	9165	9170	9175	9180	9186	1	1	2	2	3	3	4	4	5
83	9191	9196	9201	9206	9212	9217	9222	9227	9232	9238	1	1	2	2	3	3	4	4	5
84	9243	9248	9253	9258	9263	9269	9274	9279	9284	9289	1	1	2	2	3	3	4	4	5
85	9294	9299	9304	9309	9315	9320	9325	9330	9335	9340	1	1	2	2	3	3	4	4	5
86	9345	9350	9355	9360	9365	9370	9375	9380	9385	9390	1	1	2	2	3	3	4	4	5
87	9395	9400	9405	9410	9415	9420	9425	9430	9435	9440	0	1	1	2	2	3	3	4	4
88	9445	9450	9455	9460	9465	9469	9474	9479	9484	9489	0	1	1	2	2	3	3	4	4
89	9494	9499	9504	9509	9513	9518	9523	9528	9533	9538	0	1	1	2	2	3	3	4	4
90	9542	9547	9552	9557	9562	9566	9571	9576	9581	9586	0	1	1	2	2	3	3	4	4
91	9590	9595	9600	9605	9609	9614	9619	9624	9628	9633	0	1	1	2	2	3	3	4	4
92	9638	9643	9647	9652	9657	9661	9666	9671	9675	9680	0	1	1	2	2	3	3	4	4
93	9685	9689	9694	9699	9703	9708	9713	9717	9722	9727	0	1	1	2	2	3	3	4	4
94	9731	9736	9741	9745	9750	9754	9759	9763	9768	9773	0	1	1	2	2	3	3	4	4
95	9777	9782	9786	9791	9795	9800	9805	9809	9814	9818	0	1	1	2	2	3	3	4	4
96	9823	9827	9832	9836	9841	9845	9850	9854	9859	9863	0	1	1	2	2	3	3	4	4
97	9868	9872	9877	9881	9886	9890	9894	9899	9903	9908	0	1	1	2	2	3	3	4	4
98	9912	9917	9921	9926	9930	9934	9939	9943	9948	9952	0	1	1	2	2	3	3	4	4
99	9956	9961	9965	9969	9974	9978	9983	9987	9991	9996	0	1	1	2	2	3	3	3	4

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
·00	1000	1002	1005	1007	1009	1012	1014	1016	1019	1021	0	0	1	1	1	1	2	2	2
·01	1023	1026	1028	1030	1033	1035	1038	1040	1042	1045	0	0	1	1	1	1	2	2	2
·02	1047	1050	1052	1054	1057	1059	1062	1064	1067	1069	0	0	1	1	1	1	2	2	2
·03	1072	1074	1076	1079	1081	1084	1086	1089	1091	1094	0	0	1	1	1	1	2	2	2
·04	1096	1099	1102	1104	1107	1109	1112	1114	1117	1119	0	1	1	1	1	2	2	2	2
·05	1122	1125	1127	1130	1132	1135	1138	1140	1143	1146	0	1	1	1	1	2	2	2	2
·06	1148	1151	1153	1156	1159	1161	1164	1167	1169	1172	0	1	1	1	1	2	2	2	2
·07	1175	1178	1180	1183	1186	1189	1191	1194	1197	1199	0	1	1	1	1	2	2	2	2
·08	1202	1205	1208	1211	1213	1216	1219	1222	1225	1227	0	1	1	1	1	2	2	2	3
·09	1230	1233	1236	1239	1242	1245	1247	1250	1253	1256	0	1	1	1	1	2	2	2	3
·10	1259	1262	1265	1268	1271	1274	1276	1279	1282	1285	0	1	1	1	1	2	2	2	3
·11	1288	1291	1294	1297	1300	1303	1306	1309	1312	1315	0	1	1	1	2	2	2	2	3
·12	1318	1321	1324	1327	1330	1334	1337	1340	1343	1346	0	1	1	1	2	2	2	2	3
·13	1349	1352	1355	1358	1361	1365	1368	1371	1374	1377	0	1	1	1	2	2	2	3	3
·14	1380	1384	1387	1390	1393	1396	1400	1403	1406	1409	0	1	1	1	2	2	2	3	3
·15	1413	1416	1419	1422	1426	1429	1432	1435	1439	1442	0	1	1	1	2	2	2	3	3
·16	1445	1449	1452	1455	1459	1462	1466	1469	1472	1476	0	1	1	1	2	2	2	3	3
·17	1479	1483	1486	1489	1493	1496	1500	1503	1507	1510	0	1	1	1	2	2	2	3	3
·18	1514	1517	1521	1524	1528	1531	1535	1538	1542	1545	0	1	1	1	2	2	2	3	3
·19	1549	1552	1556	1560	1563	1567	1570	1574	1578	1581	0	1	1	1	2	2	3	3	3
·20	1585	1589	1592	1596	1600	1603	1607	1611	1614	1618	0	1	1	1	2	2	3	3	3
·21	1622	1626	1629	1633	1637	1641	1644	1648	1652	1656	0	1	1	2	2	2	3	3	3
·22	1660	1663	1667	1671	1675	1679	1683	1687	1690	1694	0	1	1	2	2	2	3	3	3
·23	1698	1702	1706	1710	1714	1718	1722	1726	1730	1734	0	1	1	2	2	2	3	3	4
·24	1738	1742	1746	1750	1754	1758	1762	1766	1770	1774	0	1	1	2	2	2	3	3	4
·25	1778	1782	1786	1791	1795	1799	1803	1807	1811	1816	0	1	1	2	2	2	3	3	4
·26	1820	1824	1828	1832	1837	1841	1845	1849	1854	1858	0	1	1	2	2	3	3	3	4
·27	1862	1866	1871	1875	1879	1884	1888	1892	1897	1901	0	1	1	2	2	3	3	3	4
·28	1905	1910	1914	1919	1923	1928	1932	1936	1941	1945	0	1	1	2	2	3	3	4	4
·29	1950	1954	1959	1963	1968	1972	1977	1982	1986	1991	0	1	1	2	2	3	3	4	4
·30	1995	2000	2004	2009	2014	2018	2023	2028	2032	2037	0	1	1	2	2	3	3	4	4
·31	2042	2046	2051	2056	2061	2065	2070	2075	2080	2084	0	1	1	2	2	3	3	4	4
·32	2089	2094	2099	2104	2109	2113	2118	2123	2128	2133	0	1	1	2	2	3	3	4	4
·33	2138	2143	2148	2153	2158	2163	2168	2173	2178	2183	0	1	1	2	2	3	3	4	4
·34	2188	2193	2198	2203	2208	2213	2218	2223	2228	2234	1	1	2	2	3	3	4	4	5
·35	2239	2244	2249	2254	2259	2265	2270	2275	2280	2286	1	1	2	2	3	3	4	4	5
·36	2291	2296	2301	2307	2312	2317	2323	2328	2333	2339	1	1	2	2	3	3	4	4	5
·37	2344	2350	2355	2360	2366	2371	2377	2382	2388	2393	1	1	2	2	3	3	4	4	5
·38	2399	2404	2410	2415	2421	2427	2432	2438	2443	2449	1	1	2	2	3	3	4	4	5
·39	2455	2460	2466	2472	2477	2483	2489	2495	2500	2506	1	1	2	2	3	3	4	5	5
·40	2512	2518	2523	2529	2535	2541	2547	2553	2559	2564	1	1	2	2	3	4	4	5	5
·41	2570	2576	2582	2588	2594	2600	2606	2612	2618	2624	1	1	2	2	3	4	4	5	5
·42	2630	2636	2642	2649	2655	2661	2667	2673	2679	2685	1	1	2	2	3	4	4	5	6
·43	2692	2698	2704	2710	2716	2723	2729	2735	2742	2748	1	1	2	3	3	4	4	5	6
·44	2754	2761	2767	2773	2780	2786	2793	2799	2805	2812	1	1	2	3	3	4	4	5	6
·45	2818	2825	2831	2838	2844	2851	2858	2864	2871	2877	1	1	2	3	3	4	5	5	6
·46	2884	2891	2897	2904	2911	2917	2924	2931	2938	2944	1	1	2	3	3	4	5	5	6
·47	2951	2958	2965	2972	2979	2985	2992	2999	3006	3013	1	1	2	3	3	4	5	5	6
·48	3020	3027	3034	3041	3048	3055	3062	3069	3076	3083	1	1	2	3	4	4	5	6	6
·49	3090	3097	3105	3112	3119	3126	3133	3141	3148	3155	1	1	2	3	4	4	5	6	6

ANTILOGARITHMS.

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
·50	3162	3170	3177	3184	3192	3199	3206	3214	3221	3228	1	1	2	3	4	4	5	6	7
·51	3236	3243	3251	3258	3266	3273	3281	3289	3296	3304	1	2	2	3	4	5	5	6	7
·52	3311	3319	3327	3334	3342	3350	3357	3365	3373	3381	1	2	2	3	4	5	5	6	7
·53	3388	3396	3404	3412	3420	3428	3436	3443	3451	3459	1	2	2	3	4	5	6	6	7
·54	3467	3475	3483	3491	3499	3508	3516	3524	3532	3540	1	2	2	3	4	5	6	6	7
·55	3548	3556	3565	3573	3581	3589	3597	3606	3614	3622	1	2	2	3	4	5	6	7	7
·56	3631	3639	3648	3656	3664	3673	3681	3690	3698	3707	1	2	3	3	4	5	6	7	8
·57	3715	3724	3733	3741	3750	3758	3767	3776	3784	3793	1	2	3	3	4	5	6	7	8
·58	3802	3811	3819	3828	3837	3846	3855	3864	3873	3882	1	2	3	4	4	5	6	7	8
·59	3890	3899	3908	3917	3926	3936	3945	3954	3963	3972	1	2	3	4	5	5	6	7	8
·60	3981	3990	3999	4009	4018	4027	4036	4046	4055	4064	1	2	3	4	5	6	6	7	8
·61	4074	4083	4093	4102	4111	4121	4130	4140	4150	4159	1	2	3	4	5	6	7	8	9
·62	4169	4178	4188	4198	4207	4217	4227	4236	4246	4256	1	2	3	4	5	6	7	8	9
·63	4266	4276	4285	4295	4305	4315	4325	4335	4345	4355	1	2	3	4	5	6	7	8	9
·64	4365	4375	4385	4395	4406	4416	4426	4436	4446	4457	1	2	3	4	5	6	7	8	9
·65	4467	4477	4487	4498	4508	4519	4529	4539	4550	4560	1	2	3	4	5	6	7	8	9
·66	4571	4581	4592	4603	4613	4624	4634	4645	4656	4667	1	2	3	4	5	6	7	9	10
·67	4677	4688	4699	4710	4721	4732	4742	4753	4764	4775	1	2	3	4	5	7	8	9	10
·68	4786	4797	4808	4819	4831	4842	4853	4864	4875	4887	1	2	3	4	6	7	8	9	10
·69	4898	4909	4920	4932	4943	4955	4966	4977	4989	5000	1	2	3	5	6	7	8	9	10
·70	5012	5023	5035	5047	5058	5070	5082	5093	5105	5117	1	2	4	5	6	7	8	9	11
·71	5129	5140	5152	5164	5176	5188	5200	5212	5224	5236	1	2	4	5	6	7	8	10	11
·72	5248	5260	5272	5284	5297	5309	5321	5333	5346	5358	1	2	4	5	6	7	9	10	11
·73	5370	5383	5395	5408	5420	5433	5445	5458	5470	5483	1	3	4	5	6	8	9	10	11
·74	5495	5508	5521	5534	5546	5559	5572	5585	5598	5610	1	3	4	5	6	8	9	10	12
·75	5623	5636	5649	5662	5675	5689	5702	5715	5728	5741	1	3	4	5	7	8	9	10	12
·76	5754	5768	5781	5794	5808	5821	5834	5848	5861	5875	1	3	4	5	7	8	9	11	12
·77	5888	5902	5916	5929	5943	5957	5970	5984	5998	6012	1	3	4	5	7	8	10	11	12
·78	6026	6039	6053	6067	6081	6095	6109	6124	6138	6152	1	3	4	6	7	8	10	11	13
·79	6166	6180	6194	6209	6223	6237	6252	6266	6281	6295	1	3	4	6	7	9	10	11	13
·80	6310	6324	6339	6353	6368	6383	6397	6412	6427	6442	1	3	4	6	7	9	10	12	13
·81	6457	6471	6486	6501	6516	6531	6546	6561	6577	6592	2	3	5	6	8	9	11	12	14
·82	6607	6622	6637	6653	6668	6683	6699	6714	6730	6745	2	3	5	6	8	9	11	12	14
·83	6761	6776	6792	6808	6823	6839	6855	6871	6887	6902	2	3	5	6	8	9	11	13	14
·84	6918	6934	6950	6966	6982	6998	7015	7031	7047	7063	2	3	5	6	8	10	11	13	15
·85	7079	7096	7112	7129	7145	7161	7178	7194	7211	7228	2	3	5	7	8	10	12	13	15
·86	7244	7261	7278	7295	7311	7328	7345	7362	7379	7396	2	3	5	7	8	10	12	13	15
·87	7413	7430	7447	7464	7482	7499	7516	7534	7551	7568	2	3	5	7	9	10	12	14	16
·88	7586	7603	7621	7638	7656	7674	7691	7709	7727	7745	2	4	5	7	9	11	12	14	16
·89	7762	7780	7798	7816	7834	7852	7870	7889	7907	7925	2	4	5	7	9	11	13	14	16
·90	7943	7962	7980	7998	8017	8035	8054	8072	8091	8110	2	4	6	7	9	11	13	15	17
·91	8128	8147	8166	8185	8204	8222	8241	8260	8279	8299	2	4	6	8	9	11	13	15	17
·92	8318	8337	8356	8375	8395	8414	8433	8453	8472	8492	2	4	6	8	10	12	14	15	17
·93	8511	8531	8551	8570	8590	8610	8630	8650	8670	8690	2	4	6	8	10	12	14	16	18
·94	8710	8730	8750	8770	8790	8810	8831	8851	8872	8892	2	4	6	8	10	12	14	16	18
·95	8913	8933	8954	8974	8995	9016	9036	9057	9078	9099	2	4	6	8	10	12	15	17	19
·96	9120	9141	9162	9183	9204	9226	9247	9268	9290	9311	2	4	6	8	11	13	15	17	19
·97	9333	9354	9376	9397	9419	9441	9462	9484	9506	9528	2	4	7	9	11	13	15	17	20
·98	9550	9572	9594	9616	9638	9661	9683	9705	9727	9750	2	4	7	9	11	13	16	18	20
·99	9772	9795	9817	9840	9863	9886	9908	9931	9954	9977	2	5	7	9	11	14	16	18	20

Angle.		Chord.	Sine.	Tangent.	Co-tangent.	Cosine.			
Degrees.	Radians.								
0°	0	000	0	0	∞	1	1·414	1·5708	90°
1	·0175	·017	·0175	·0175	57·2900	·9998	1·402	1·5533	89
2	·0349	·035	·0349	·0349	28·6363	·9994	1·389	1·5359	88
3	·0524	·052	·0523	·0524	19·0811	·9986	1·377	1·5184	87
4	·0698	·070	·0698	·0699	14·3007	·9976	1·364	1·5010	86
5	·0873	·087	·0872	·0875	11·4301	·9962	1·351	1·4835	85
6	·1047	·105	·1045	·1051	9·5144	·9945	1·338	1·4661	84
7	·1222	·122	·1219	·1228	8·1443	·9925	1·325	1·4486	83
8	·1396	·140	·1392	·1405	7·1154	·9903	1·312	1·4312	82
9	·1571	·157	·1564	·1584	6·3138	·9877	1·299	1·4137	81
10	·1745	·174	·1736	·1763	5·6713	·9848	1·286	1·3963	80
11	·1920	·192	·1908	·1944	5·1446	·9816	1·272	1·3788	79
12	·2094	·209	·2079	·2126	4·7046	·9781	1·259	1·3614	78
13	·2269	·226	·2250	·2309	4·3315	·9744	1·245	1·3439	77
14	·2443	·244	·2419	·2493	4·0108	·9703	1·231	1·3265	76
15	·2618	·261	·2588	·2679	3·7321	·9659	1·218	1·3090	75
16	·2793	·278	·2756	·2867	3·4874	·9613	1·204	1·2915	74
17	·2967	·296	·2924	·3057	3·2709	·9563	1·190	1·2741	73
18	·3142	·313	·3090	·3249	3·0777	·9511	1·176	1·2566	72
19	·3316	·330	·3256	·3443	2·9042	·9455	1·161	1·2392	71
20	·3491	·347	·3420	·3640	2·7475	·9397	1·147	1·2217	70
21	·3665	·364	·3584	·3839	2·6051	·9336	1·133	1·2043	69
22	·3840	·382	·3746	·4040	2·4751	·9272	1·118	1·1868	68
23	·4014	·399	·3907	·4245	2·3559	·9205	1·104	1·1694	67
24	·4189	·416	·4067	·4452	2·2460	·9135	1·089	1·1519	66
25	·4363	·433	·4226	·4663	2·1445	·9063	1·075	1·1345	65
26	·4538	·450	·4384	·4877	2·0503	·8988	1·060	1·1170	64
27	·4712	·467	·4540	·5095	1·9626	·8910	1·045	1·0996	63
28	·4887	·484	·4695	·5317	1·8807	·8829	1·030	1·0821	62
29	·5061	·501	·4848	·5543	1·8040	·8746	1·015	1·0647	61
30	·5236	·518	·5000	·5774	1·7321	·8660	1·000	1·0472	60
31	·5411	·534	·5150	·6009	1·6643	·8572	·985	1·0297	59
32	·5585	·551	·5299	·6249	1·6003	·8480	·970	1·0123	58
33	·5760	·568	·5446	·6494	1·5399	·8387	·954	·9948	57
34	·5934	·585	·5592	·6745	1·4826	·8290	·939	·9774	56
35	·6109	·601	·5736	·7002	1·4281	·8192	·923	·9599	55
36	·6283	·618	·5878	·7265	1·3764	·8090	·908	·9425	54
37	·6458	·635	·6018	·7536	1·3270	·7986	·892	·9250	53
38	·6632	·651	·6157	·7813	1·2799	·7880	·877	·9076	52
39	·6807	·668	·6293	·8098	1·2349	·7771	·861	·8901	51
40	·6981	·684	·6428	·8391	1·1918	·7660	·845	·8727	50
41	·7156	·700	·6561	·8693	1·1504	·7547	·829	·8552	49
42	·7330	·717	·6691	·9004	1·1106	·7431	·813	·8378	48
43	·7505	·733	·6820	·9325	1·0724	·7314	·797	·8203	47
44	·7679	·749	·6947	·9657	1·0355	·7193	·781	·8029	46
45°	·7854	·765	·7071	1·0000	1·0000	·7071	·765	·7854	45°
			Cosine.	Co-tangent.	Tangent.	Sine.	Chord.	Radians.	Degrees.
								Angle.	

ANSWERS

EXERCISES II, p. 21

4. 15 dynes.
5. 69·6 dynes.
6. (*a*) 910 C.G.S. units.
   (*b*) 788 C.G.S. units.
7. 0·0131 gauss, 0·0128 gauss.
8. 0·00381 gauss, 0·00391 gauss.
10. 0·160 gauss.
11. 0·461 gauss.
12. 31·7 C.G.S. units.
13. 472 C.G.S. units.
14. 13·9.

EXERCISES III, p. 34

4. 471 C.G.S. units.
5. 0·249 C.G.S. unit.
6. 0·226 or 0·628 gauss.
7. 0·0744 C.G.S. unit.
10. 0·881 C.G.S. unit.
14. 19° 56′.

EXERCISES IV, p. 52

4. (*a*) 20·8 dynes, (*b*) 4.17 dynes.
5. 3 C.G.S. units.
9. 9 C.G.S. units.
18. 5·20 C.G.S. units.
19. 15·7 ergs.
20. 75 ergs.

EXERCISES V, p. 72

2. From A to B.
3. 20100 C.G.S. units.
4. 2640 C.G.S. units.
5. 0·05 C.G.S. units, 3·3 ergs.
7. 21·2 C.G.S. units.
10. 15·4 ergs and 2·37 ergs.
11. 4·65 C.G.S. units, 3·22 C.G.S. units.
12. 3750 ergs.
13. 300 C.G.S. units, 15 C.G.S. units.
14. From B to A, −0·716 C.G.S. unit.
15. 5·74.
16. $7{\cdot}5 \times 10^{-4}$ E.M.U.

EXERCISES VI, p. 86

1. $2{\cdot}42 \times 10^4$ ergs.
2. $9{\cdot}3 \times 10^7$ ergs per sec.
5. 2·63 ohms, 0·285 amp.
6. 80·2 ohms.
7. 1·03 volt.
8. 1·65 ohm.
9. 180 volts.
10. (*a*) 1·55 amp., (*b*) 0·75 amp.
11. (*a*) 0·0395 amp., (*b*) 0·0100 amp.
12. 13·4 calories, 2·06 calories.
13. 53·1 m.
14. 0·152 volt.
15. 2020 ohms.
16. (*a*) 2·70 calories.
    (*b*) 0·252 calorie.
17. 0·316 amp.
18. 5 approx.
19. 4 ohms.
20. All in series with $3\frac{1}{3}$ ohms.

EXERCISES VII, p. 111

2. 0·00333 ohm.
3. 2990 ohms in series.
8. $1{\cdot}72 \times 10^{-6}$ pr. C.G.S. unit.
12. 36° C.
13. 1·11.
15. −0·062.
17. 0·337 ohm.
18. 0·00169.

Exercises VIII, p. 131

4. 3·97 amp.
6. 0·257 gauss.
7. −0·019
16. 9·05 gm.

Exercises IX, p. 152

1. 30 dynes.
3. 35·4 dyne-cm.
6. 90·5 dynes.
9. 10^{-2} volts.
10. 0·256 volt.
11. $1{\cdot}005 \times 10^{-2}$ henry.
15. $1{\cdot}33 \times 10^{-2}$ henry.
17. $1{\cdot}69 \times 10^{-2}$ henry.

Exercises X, p. 166

2. 0·302 volt, 0·213 volt.
7. 0·795 amp.
10. 35·4 amps.
11. 11·0 millihenries.
12. 33° 33′.

Exercises XI, p. 173

8. 5·76 volts.

Exercises XII, p. 186

3. $1{\cdot}59 \times 10^{4}$.
4. $3{\cdot}95 \times 10^{4}$ metres.
9. $8{\cdot}13 \times 10^{5}$.

Exercises XV, p. 213

10. (*a*) 41·3 calories, (*b*) 96·3 calories.

Exercises XVI, p. 224

3. 0·494 gauss.
   0·526 gauss.
4. 0·344.
11. 73° 54′.

Exercises XVII, p 237

2. 90·5 C.G.S. units
3. 0·0222 gauss.
6. 126·7.
7. 1·000276, 0·999834.

INDEX

THE END

PRINTED IN ENGLAND BY WILLIAM CLOWES AND SONS, LIMITED, BECCLES.

PLEASE DO NOT REMOVE
CARDS OR SLIPS FROM THIS POCKET

UNIVERSITY OF TORONTO LIBRARY

SIGMUND SAMUEL LIBRARY